AMSAA RELIABILITY GROWTH HANDBOOK

1. INTRODUCTION

1.1 Foreword. This handbook provides methodology and concepts to assist in reliability growth planning and a structured approach for reliability growth assessments. The planning aspects, which are covered in section 2 of this handbook, address the planned growth curve and related milestones. The assessment techniques, which are designed to realistically evaluate reliability in the presence of a changing configuration, are based on demonstrated and projected values and are covered in sections 3 and 4, respectively. The material in this handbook updates MIL-HDBK-189 [1].

1.1.1 Why. Reliability growth management procedures were developed to help guide the materiel acquisition process for new military systems. This process is usually complex and difficult for many reasons. Generally, these systems require new technologies and represent a challenge to the state of the art. Moreover, the requirements for reliability, maintainability and other performance parameters are usually highly demanding. Consequently, striving to meet these requirements represents a significant portion of the entire acquisition process and, as a result, the setting of priorities and the allocation and reallocation of resources such as funds, manpower and time are often formidable management tasks.

1.1.2 What. Reliability growth management procedures address the priorities and allocation problem. These techniques will enable the manager to plan, evaluate and control the reliability of a system during its development stage. The reliability growth concepts and methodologies presented in this handbook have evolved over the last couple of decades by actual applications to Army, Navy and Air Force systems. Through these applications reliability growth management technology has been developed to the point where considerable payoffs resulting from the effective management of attaining system reliability can now be achieved.

1.1.3 Layout. This handbook is written for both the manager and the analyst. Generally, the further into the handbook one reads, the more technical and detailed the material becomes. The fundamental concepts are covered early in the handbook and the details regarding the implementation of these concepts are discussed primarily in the latter sections. This format, together with an objective for as much completeness as possible within each section, have resulted in some concepts being repeated or discussed in more than one place in the handbook. This should help facilitate the use of this handbook for studying certain topics without extensively referring to previous material.

1.2 Scope.

1.2.1 Purpose. This handbook provides an understanding of the concepts and principles of reliability growth, advantages of managing reliability growth, and guidelines and procedures to be used in managing reliability growth. It should be noted that this handbook is not intended to serve as a reliability growth plan to be applied to a program without any tailoring. This handbook, when used in conjunction with knowledge of the system and its development program, will allow the development of a reliability growth management plan that will aid in developing a final system that meets its requirements and lowers the life cycle cost of the fielded systems.

1.2.2 Application. This handbook is intended for use on systems/equipment during their development phase by both producer and customer personnel.

1.3 Definition of Terms.

1.3.1 Reliability. Reliability is the probability that an item will perform its intended function for a specified interval under stated conditions. The term "specified interval" refers to the length of the mission as described in a mission profile. The term "stated conditions" refers to the complete definition of the scenario in which the system will operate. These conditions should reflect operational usage.

1.3.2 Reliability Growth. Reliability growth is the improvement in a reliability parameter over a period of time due to changes in product design or the manufacturing process.

1.3.3 Reliability Growth Management. Reliability growth management is the systematic planning for reliability achievement as a function of time and other resources, and controlling the ongoing rate of achievement by reallocation of resources based on comparisons between planned and assessed reliability values.

1.3.4 Repair. A repair is the replacement of a failed item with an "identical" item in order to return the item to its mission.

1.3.5 Fix. A fix is a corrective action that results in a change to the design or to the manufacturing process of the item for the purpose of improving its reliability.

1.4 Overview.

1.4.1 Benefits of Reliability Growth Management. The following benefits can be realized by the utilization of reliability growth management.

1.4.1.1 Finding Unforeseen Deficiencies. The initial prototypes for a complex system with major technological advances will invariably have significant reliability and performance deficiencies that could not be foreseen in the early design stage. This is also true of prototypes that are "simply" the integration of existing systems. Unforeseen problems are the norm in achieving seamless interoperation and interfacing between already developed systems. Reliability growth testing will surface these deficiencies.

1.4.1.2 Designing in Improvement through Surfaced Problems. Even if some potential problems can be foreseen, their significance might not. Prototypes are subjected to a development-testing program to surface those problems that drive the failure rate so that the necessary improvements in system design can be made. The ensuing system reliability and performance characteristics will depend on the number and effectiveness of these fixes. The ultimate goal of the development test program is to meet the system reliability and performance requirements.

1.4.1.3 Reducing the Risk of Final Demonstration. Experience has shown that programs that rely simply on a final demonstration by itself to determine compliance with the reliability requirements do not, in many cases, achieve the reliability objectives within the allocated resources. Emphasis on reliability performance prior to the final demonstration using quantitative reliability growth could substantially increase the chance of passing, or even replace a final demonstration.

1.4.1.4 Increasing the Probability of Meeting Objectives. This can be achieved by setting interim reliability goals to be met during the development testing program and the necessary allocation and reallocation of resources to attain these goals. A comprehensive approach to reliability growth management throughout the development program organizes this process.

1.4.2 Sketch of Reliability Growth Management. The essence of reliability growth management consists of planning, evaluating and controlling the growth process.

1.4.2.1 Reliability Growth Planning. Reliability growth planning addresses program schedules, amount of testing, resources available and the realism of the test program in achieving the requirements. The planning is quantified and reflected in the construction of a reliability growth program plan curve. This curve establishes interim reliability goals throughout the program.

1.4.2.2 Reliability Growth Assessment. To achieve these goals it is important that the program manager be aware of reliability problems during the conduct of the program so that he can effect whatever changes are necessary, e.g., increased reliability emphasis. It is, therefore, essential that periodic assessments of reliability be made during the test program (usually at the end of a test phase) and compared to the planned reliability growth values.

1.4.2.3 Controlling Reliability Growth. These assessments provide visibility of achievements and focus on deficiencies while there is still time to affect the system design. By making appropriate decisions with regard to the timely incorporation of effective fixes into the system commensurate with attaining the milestones and requirements, management can control the growth process.

1.4.3 Management's Role. The various techniques associated with reliability growth management do not, in themselves, manage. They simply make reliability a more visible and manageable characteristic. Every level of management can take advantage of this visibility by requesting reliability growth plans and progress handbooks for review. Without this implementation, reliability growth cannot truly be managed.

The planned growth curve and milestones are only targets. They do not imply that reliability will automatically grow to these values. On the contrary, these values will be attained only with the incorporation of an adequate number of effective design fixes into the system. This requires dedicated management attention to reliability growth. The methods in this handbook are for the purpose of assisting management in making timely and appropriate decisions to ensure sufficient support of the reliability engineering design effort throughout the development testing program.

High level management of reliability growth is necessary in order to have available all the options for difficult program decisions. For example, high level decisions in the following areas may be necessary in order to ensure that reliability goals are achieved:

- Revise the program schedule.
- Increase testing.
- Fund additional development efforts.
- Add or reallocate program resources.
- Stop the program until interim reliability goals have been demonstrated.

Although some of these options may result in severe program delay or significant increase in development costs, they may have to be exercised in order to field equipment that meets user needs and has acceptable total life cycle costs.

1.4.4 Basic Reliability Activities. Reliability growth management is part of the system engineering process. It does not take the place of the other basic reliability program activities such as:

- Design predictions
- Apportionment
- Failure modes and effects analysis
- Stress analysis

Instead, reliability growth management provides a means of viewing all the reliability program activities in an integrated manner.

1.4.5 Reliability Growth Process.

1.4.5.1 Basic Process. Reliability growth is the result of an iterative design process. As the design matures, it is investigated to identify actual or potential sources of failures. Further design effort is then spent on these problem areas. The design effort can be applied to either product design or manufacturing process design. The iterative process can be visualized as a simple feedback loop as in Figure 1. This illustrates that there are three essential elements involved in achieving reliability growth:

- Detection of failure sources,
- Feedback of problems identified and
- Redesign effort based on problems identified.

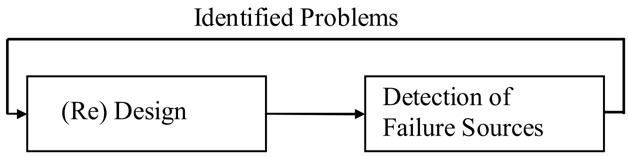

Figure 1. Reliability Growth Feedback Model.

Furthermore, if failure sources are detected by testing, a fourth element is necessary:

- Fabrication of hardware.

And, following redesign, detection of failure sources serves as:

- Verification of redesign effort.

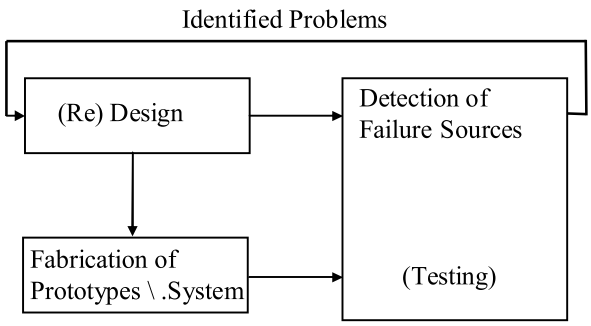

Figure 2. Reliability Growth Feedback Model with Hardware.

1.4.5.2 Growth Rate. The rate at which reliability grows is dependent on:

- how rapidly activities in this loop can be accomplished,
- how significant the identified problems are, and

- how well the redesign effort solves the identified problems without introducing new problems.

Any of these activities may act as a bottleneck. The cause and degree of the bottleneck may vary from one development program to the next, and even within a single program may vary from one stage of development to the next.

1.4.6 Reliability Growth Management Control Processes. Figures 1, 2, 3, and 5 illustrate the growth process and associated management processes in a skeleton form. This type of illustration is used so that the universal features of these processes may be addressed. The representation of an actual program or program phase may be considerably more detailed. This detailing may include specific inputs to, and outputs from, the growth process, additional activity blocks, and more explicit decision logic blocks.

1.4.6.1 Basic Methods. There are two basic ways that the manager evaluates the reliability growth process. The first method is to utilize assessments (quantitative evaluations of the current reliability status) that are based on information from the detection of failure sources. The second method is to monitor the various activities in the process to assure himself that the activities are being accomplished in a timely manner and that the level of effort and quality of work are in compliance with the program plan. Each of these methods complements the other in controlling the growth process.

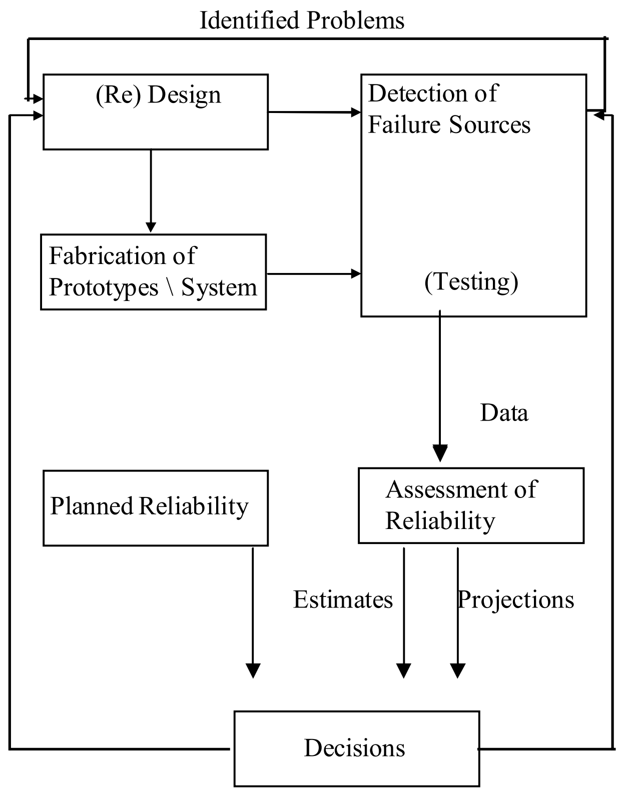

Figure 3. Reliability Growth Management Model (Assessment).

1.4.6.2 Comparison of Methods. The assessment approach is results oriented; however, the monitoring approach, which is activities oriented, is used to supplement the assessments and may have to be relied on entirely early in a program. This is often necessary because of the lack of sufficient objective information in the early program stages.

1.4.6.3 Assessment. Figure 3 illustrates how assessments may be used in controlling the growth process. Reliability growth management differs from conventional reliability program management in two major ways. First, there is a more objectively developed growth standard against which assessments are compared. Second, the assessment methods used can provide more accurate evaluations of the reliability of the present equipment configuration. A comparison between the assessment and the planned value will suggest whether the program is progressing as planned, better than planned, or not as well as planned. If the progress is falling short, new strategies should be developed. These strategies may involve the reassignment of resources to work on identified problem areas or may result in adjustment of the timeframe or a re-examination of the validity of the requirement. Figure 4 illustrates an example of both the planned reliability growth and assessments.

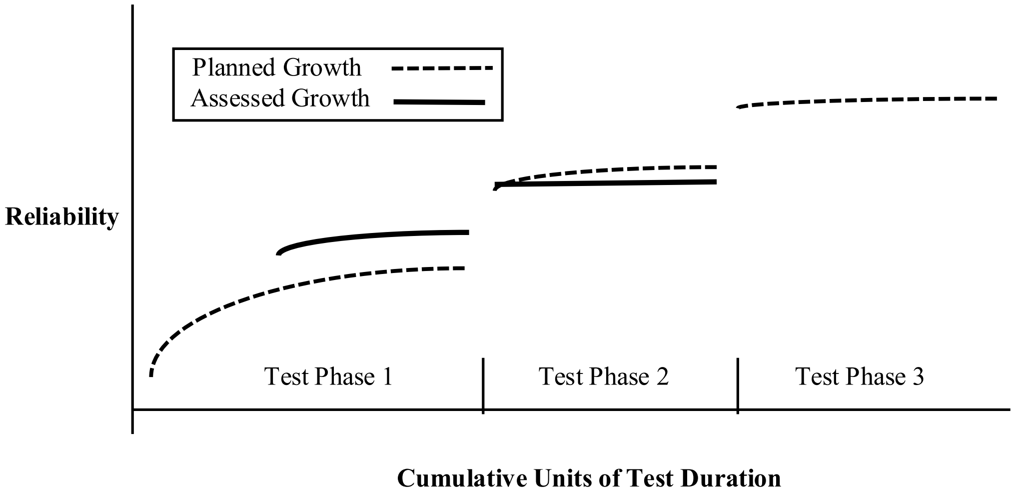

Figure 4. Example of Planned Growth and Assessments.

1.4.6.4 Monitoring. Figure 5 illustrates control of the growth process by monitoring the growth activities. Since there is no simple way to evaluate the performance of the activities involved, management based on monitoring is less definitive than management based on assessments. Nevertheless, this activity is a valuable complement to reliability assessments for a comprehensive approach to reliability growth management. But standards for level of effort and quality of work accomplishment must, of necessity, rely heavily on the technical judgment of the evaluator. Monitoring is intended to assure that the activities have been performed within schedule and meet appropriate standards of engineering practice. It is not intended to second-guess the designer, e.g., redo his stress calculations. One of the better examples of a monitoring activity is the design review. The design review is a planned monitoring of a product design to assure that it will meet the performance requirements during operational use. Such reviews of the design effort serve to determine the progress being made in achieving the design objectives. Perhaps the most significant aspect of the design review is its emphasis on technical judgment, in addition to quantitative assessments of progress.

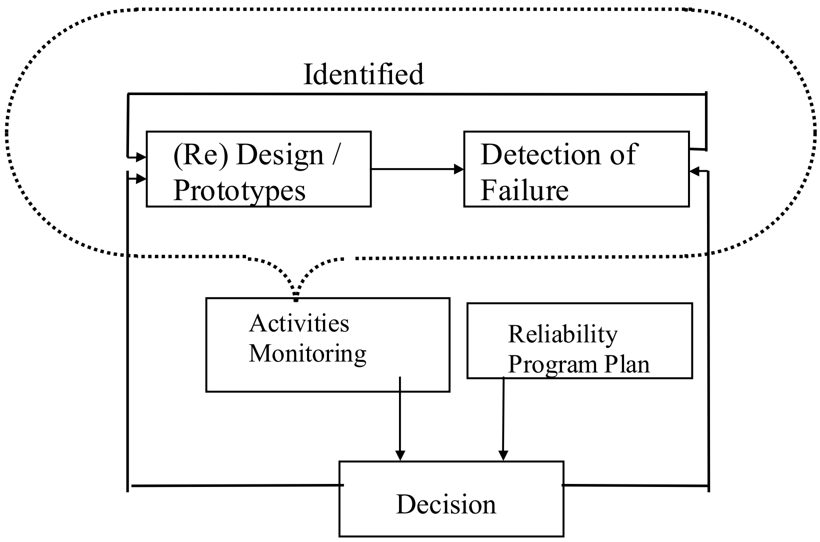

Figure 5. Reliability Growth Management Model (Monitoring).

1.4.7 Factors Influencing Growth Curve Shape. This section introduces factors that affect the shape of the growth curve. Such things as the current stage of the development program, the current test phase, the system configuration under test, the timing of design change insertion, and the units of measure for test duration all influence the growth curve's shape.

1.4.7.1 Stages of the Development Program. Generally, any system development program is divided into stages having different objectives for each stage. The names and objectives for each stage in a given development program need not be the ones given here. These stages are given as representative of a typical development:

- Proposal. There is no hardware at this stage. This is the engineering and accounting paper analysis of differing proposed solutions and designs. In this stage the concern is over what are the requirements, can they be met, and if so, how and at what estimated cost?

- Conceptual. Experimental prototypes are built at this stage. These may bear little resemblance to the actual system. They are for proof-of-principle.

- Validation. Prototypes much like the final system are built and tested. This stage tries to achieve the performance and reliability objectives for the system.

- Full Scale Development. Systems built as though they were in production are tested to work out final design details and manufacturing procedures.

Quantitative reliability growth management can be used during the validation and full-scale development stages of the program. It could be argued that the different nature of the testing going on in these stages is different enough to cause different rates of growth to occur. How much different the types of testing are determines how they will be treated in creating the planning growth curve. We will discuss this further in Section 1.4.7.6.

1.4.7.2 Test Phases. Within a development stage it is quite likely that testing will be broken up into alternating time periods of active testing followed by none. Each period of active testing can be viewed as a testing phase. Also, within a development stage it is quite likely that more than one type of testing will be going on (e.g., performance testing). If these other tests that are not specifically for reliability follow the intended operating environment and the intended use stresses well enough, and if design changes are made on the basis of these tests, then the information gathered may be incorporated into the reliability growth test data base. These would also be called reliability growth testing phases. It is to be expected that the reliability will grow from one phase to the next. The reliability growth planning curve should reflect this.

1.4.7.3 System Configurations. In an absolute sense, any change to the design of a system constitutes a new configuration. For our purposes, we will term a specific design a new configuration if there has been one significant design change, or enough little design changes, that cause an obviously different failure rate for the system. It is possible that two or more testing phases could be grouped together for analysis based on the configuration tested in these phases being substantially unchanged. It is also possible that one design change is so effective at increasing reliability that a new configuration could occur within a test phase. System configuration decisions can also be made on the basis of engineering judgement. Obviously, the configuration under test has great influence on the growth curve.

1.4.7.4 Timing of Fixes. The replacement of a part with another part identical to the first is termed a repair. Replacing, or eliminating, a part due to a design change is termed a fix. Fixes are intended to reduce the rate at which the system fails. Repairs make no change in the failure rate of the system. The time of insertion of a fix affects the pattern of reliability growth.

1.4.7.4.1 Test-Fix-Test. In an absolutely pure test-fix-test program, when a failure is observed, testing stops until a design change is implemented on the system under test. When the testing resumes, it is with a system that has incrementally better reliability. The graph of reliability for this testing strategy is a series of small increasing steps, with each step stretching out longer to represent a longer time between failures. Such a graph can be approximated by a smooth curve. See Figure 6.

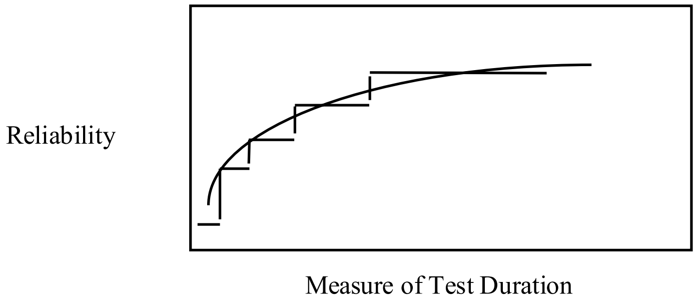

Figure 6. Graph of Reliability in a Test-Fix-Test Program.

Such a pure test-fix-test program is impractical in most situations. Testing is likely to continue with a repair, and the fix will be implemented later. Nevertheless, if fixes are inserted as soon as possible and while testing is still proceeding, the stair step like reliability increases and the shape of the approximating curve and will be similar, but rise at a slower rate. This is due to the reliability remaining at the same level that it was at when the failure happened until the fix is inserted. Thus the steps will all be of longer length, but the same height. Continuing to test after the fix is inserted will serve to verify the goodness of the design change.

1.4.7.4.2 Test-Find-Test. During a test-find-test program the system is also tested to determine problem failure modes. However, unlike the test-fix-test program, fixes are not incorporated into the system during the test. Rather, the fixes are all inserted into the system at the end of the test phase and before the next testing period. Since a large number of fixes will generally be incorporated into the system at the same time, there is usually a significant jump in system reliability at the end of the test phase. The fixes incorporated into the system between test phases are called delayed fixes. See Figure 7.

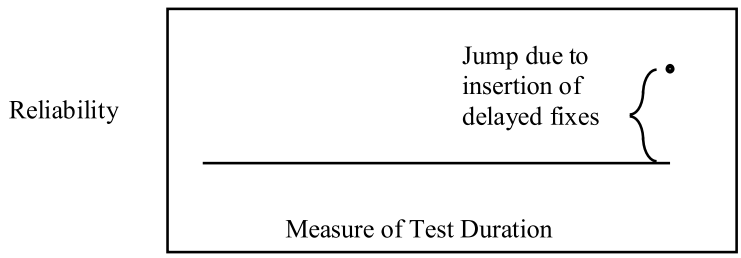

Figure 7. Graph of Reliability in a Test-Find-Test Program.

1.4.7.4.3 Test-Fix-Test with Delayed Fixes. The test program commonly used in development testing employs a combination of the two types of fix insertions discussed above. In this case, some fixes are incorporated into the system during the test while other fixes are delayed until the end of the test phase. Consequently, the system reliability will generally be seen as a smooth process during the test phase and then jump due to the insertion of the delayed fixes. See Figure 8.

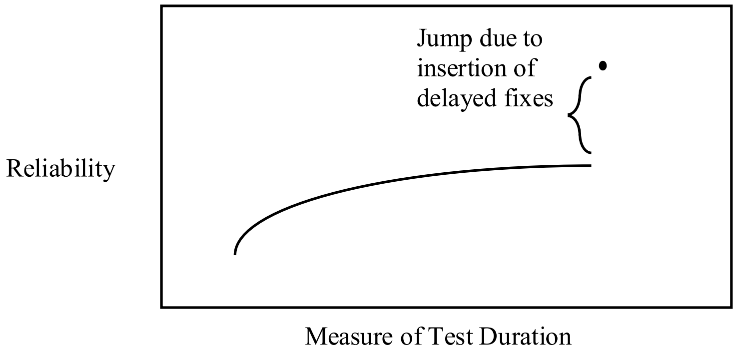

Figure 8. Graph of Reliability in a Test-Fix-Test Program with Delayed Fixes.

1.4.7.5 Combined Influences of Factors on Reliability Growth Curve Shape. In order to reach the goal reliability, the development-testing program will usually consist of several major test phases. Within each test phase the fix insertion may be carried out in any one of the three ways discussed above. As an example, suppose that testing were conducted during the validation and full-scale development stages of the program. Each stage would have at least one major test phase, implying a minimum of two major test phases for the program. In this case, there would be $3^2 = 9$ general ways the reliability may grow during the development test. See Figure 9.

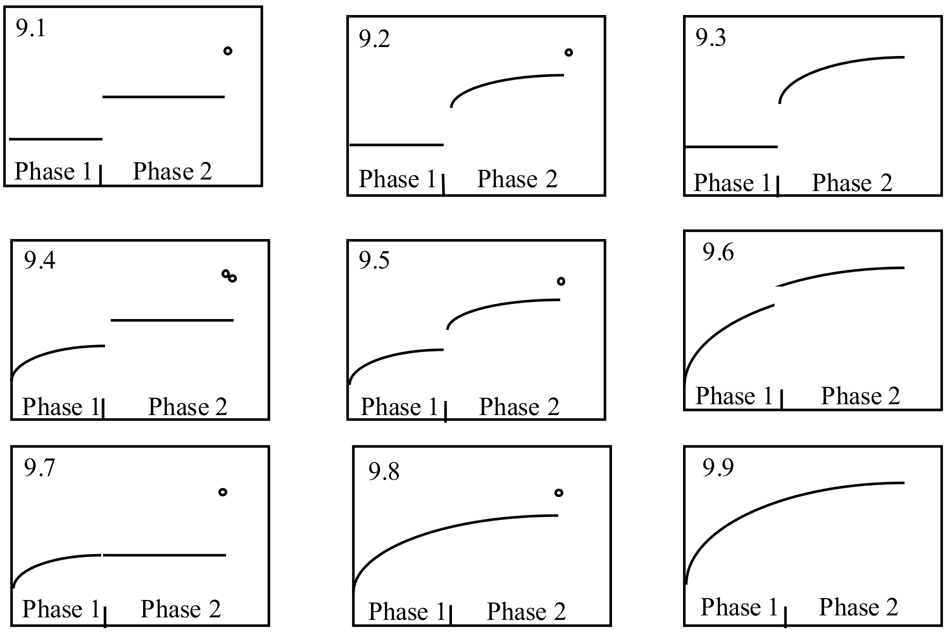

Figure 9. The Nine Possible General Growth Patterns for Two Test Phases.

Row 1 shows Phase 1 as having all fixes delayed until the end of the testing phase. Row 2 shows Phase 1 as having some fixes inserted during test and some delayed. Row 3 shows Phase 1 as having all fixes inserted during test, with none delayed. Column 1 shows Phase 2 as having all fixes delayed until the end of the testing phase. Column 2 shows Phase 2 as having some fixes inserted during test and some delayed. Column 3 shows Phase 2 as having all fixes inserted during test, with none delayed.

Figures 9.1 and 9.9 represent the two extremes in possible growth test patterns. There are some distinct statistical advantages to following a complete test-fix-test program:

- The estimated value of reliability at any point along the smooth growth curve is an instantaneous value. That is, it is not dragged down by averaging with the failures that accrued due to earlier (and hopefully) less reliable configurations.

- Confidence limits about the true value of reliability can be established.

- While the impact of the jumps in reliability can be assessed using a mix of some engineering judgement (this will be discussed in the section on Reliability Growth Projection) and direct calculation, the estimate of reliability in a test-fix-test program is based solely on data.

- In a test-fix-test program, the goodness of the design changes is continuously being assessed in the estimate of reliability.

A development stage may consist of more than one distinct test phase. For example, suppose that testing is stopped part way through the full-scale development stage, and delayed fixes are incorporated into the system. The testing in this case may be considered as two major test phases during this stage, giving three phases for the whole program. If a program had three major test phases then there would be $3^3 = 27$ patterns of reliability growth. Obviously this manner of determining the possible number of growth patterns can be extended to any number of phases.

1.4.7.6 Growth Curve Reinitialization. The differences in the growth curves between phases shown in Figures 9.5 and 9.6 represent the difference mentioned in the last paragraph of Section 1.4.7.1. Underlying Figure 9.6 is the assumption that the testing environment and engineering efforts are the same across test phases, thus the continuation of the same growth curve into the succeeding phase, after the jump for delayed fixes. In Figure 9.5 some factor influencing the rate of growth has substantially changed between the phases, which is reflected in a new growth curve for the succeeding phase. This is called reinitializing the growth curve. It must be emphasized that reinitialization of a growth curve is only justified if the testing environment is so different as to introduce a new set of failure modes, or the engineering effort is so different as to be best represented as a totally new program.

1.4.7.7 Shape Changes Due to Calendar Time. Reliability growth is often depicted as a function of test time for evaluation purposes. For management and presentation purposes it may be desirable to portray reliability growth as a function of calendar time. This can be accomplished by determining the number of units of test duration that will have been completed at each measure point in calendar time and then plotting the value that corresponds to the completed test duration above that calendar point. This is a direct function of the program schedule. Figure 10 shows the reliability growth of a system as a function of test time and calendar time.

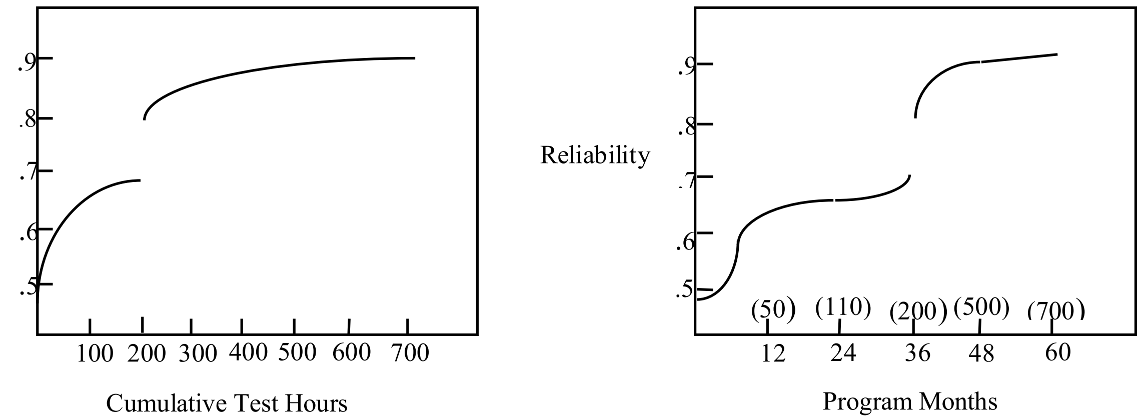

Figure 10. Comparison of Growth Curves Based on Test Duration Vs Calendar Time.

1.4.8 Reliability Growth Concepts.

1.4.8.1 Levels of Consideration for Growth. Planning and controlling reliability growth can be divided as to levels of consideration along both a program basis and an item under test basis.

- Program considerations:
    - Global: This approach treats reliability growth on a total basis over the entire development program.
    - Local: The other approach treats reliability growth on a phase-by-phase basis.
- Item Under Test considerations:
    - System Level: The entire system as it is intended to be fielded is tested.

- Subsystem Level: The obvious meaning is the testing of a major and reasonably complex component of the whole system (e.g., an engine for a vehicle). Sometimes, the subsystem would seem to be an autonomous unit, but because the requirement is for this unit to operate in conjunction with other units to achieve an overall functional goal it is really only part of "the system" (e.g., radar for an air defense system).

The appropriate level of consideration can be different at different times within the development.

1.4.8.2 Analysis of Previous Programs. Analysis of previous similar programs is used to develop guidelines for predicting the growth during future programs. Such analysis may be performed on either overall programs or individual program phases, or both. Of particular interest are the patterns of growth observed and the effect of program characteristics on initial values and growth rates. The U.S. Army Materiel Systems Analysis Activity (AMSAA) has conducted a data study, [2], that is a useful guide in choosing appropriate growth rates for various system types.

1.4.9 Planning.

1.4.9.1 Planned Growth Curve. The planned growth curve is a picture of the anticipated reliability growth for the entire program. It is an essential part of the reliability growth management methodology and is important to any reliability program. This curve is constructed early in the development program generally before hard reliability data are obtained and is typically a joint effort between the program manager and contractor. Its primary purpose is to provide management with guidelines as to what reliability can be expected at any stage of the program and to provide a basis for evaluating the actual progress of the reliability program based upon generated reliability data. The planned growth curve can be constructed on a phase-by-phase basis. See Figure 11.

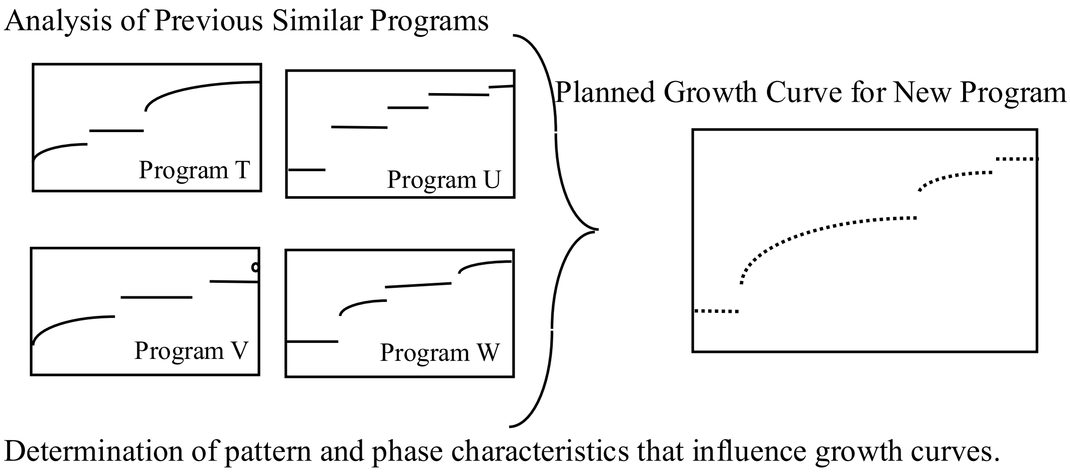

Figure 11. Development of Planned Growth Curve on a Phase by Phase Basis.

1.4.9.2 **Idealized Growth Curve.** An Idealized Growth Curve is a planned growth curve that consists of a single smooth curve based on initial conditions, an assumed growth rate, and/or planned management strategy. This curve is a strict mathematical function of the input parameters across the measure of test duration (e.g., time, distance, trials), thus the name "Idealized." No program can be expected to assume this exact mathematical ideal shape, but it is useful in setting interim goals. See Figure 12.

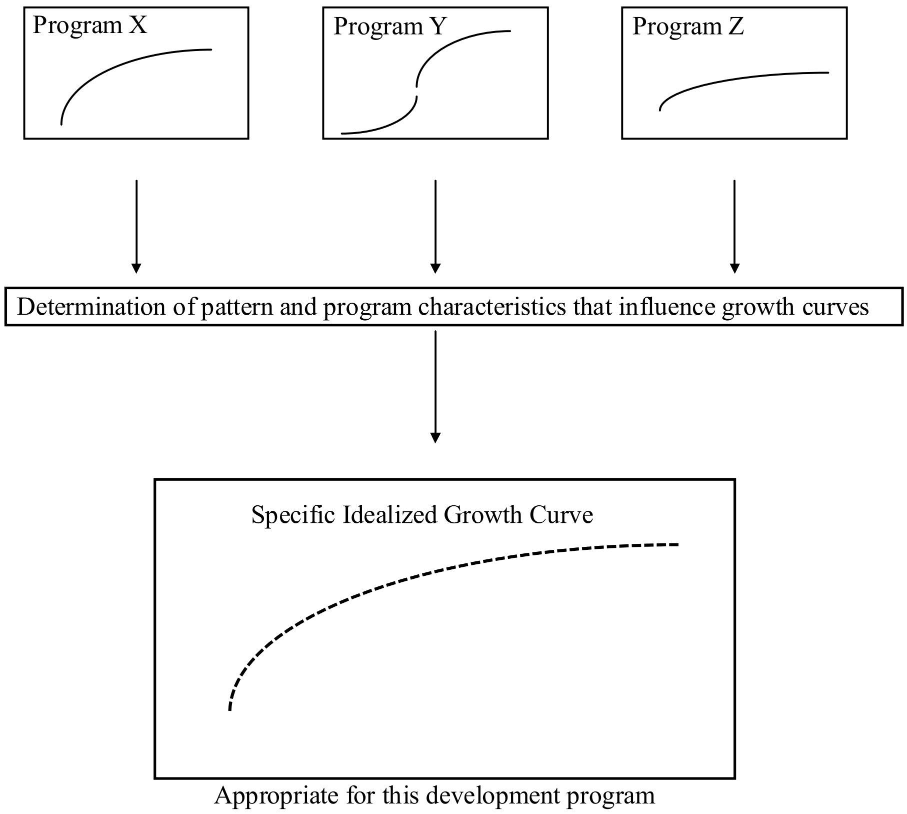

Figure 12. Global Analysis Determination of Planned Growth Curve.

1.4.10 Tracking.

1.4.10.1 Demonstrated Reliability. A demonstrated reliability value is based on actual test data and is an estimate of the current attained reliability. The assessment is made on the system configuration currently undergoing test, not on an anticipated configuration, nor a prior configuration. This number allows for the effects of even recently introduced fixes into the system as its calculation incorporates the trend of growth established over the history, to date, of the development program.

1.4.10.2 Reliability Growth Tracking Curve. The reliability growth tracking curve is the curve that best fits the data being analyzed. It can be based on data solely within one phase or data from several phases. Whatever period of testing is used to form a database, this curve is the statistical best representation from a family of growth curves of the overall reliability growth of the system. It depicts the trend of growth that has been established over the database. Thus, if the database covers the entire program to date, the right end point of this curve is the current demonstrated reliability. See Figure 13.

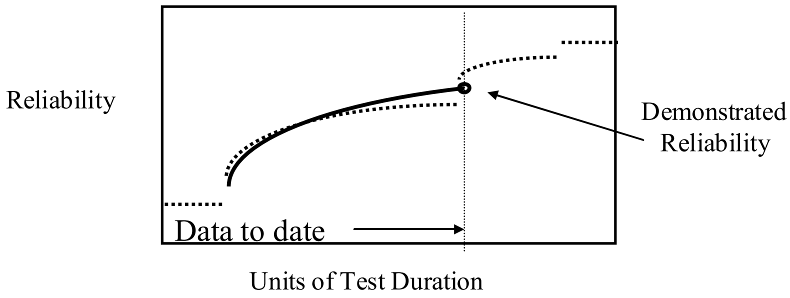

Figure 13. Reliability Growth Tracking Curve.

1.4.11 Projection.

1.4.11.1 Extrapolated Reliability. Extrapolating a growth curve beyond the currently available data shows what reliability a program can be expected to achieve, as a function of additional test duration, provided the conditions of test and the engineering effort to improve reliability are maintained at their present levels (i.e., the established trend continues).

1.4.11.2 Projected Reliability. A reliability projection is an assessment of reliability that can be anticipated at some future point in the development program. The projection is based on the achievement to date and engineering assessments of future program characteristics. Projection is a particularly valuable analysis tool when a program is experiencing difficulties because it enables investigation of program alternatives.

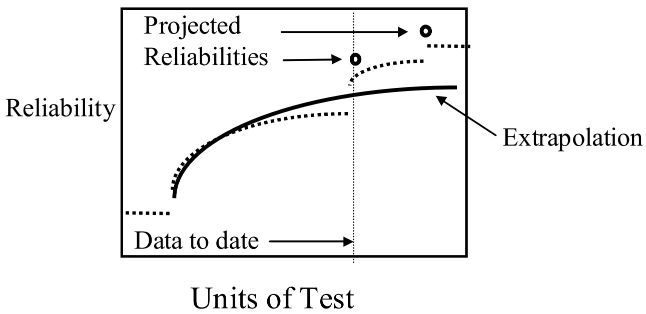

Figure 14. Extrapolated and Projected Reliabilities.

REFERENCES

1. MIL-HDBK-189, Reliability Growth Management, 13 February 1981

2. Ellner, Paul M. and Trapnell, Bruce, AMSAA Interim Note IN-R-184, AMSAA Reliability Growth Data Study, January 1990

APPENDIX A

Background

Before going into the specifics of devising reliability growth planning curves, it is useful to look at the history of this process to learn why the curves have the form that they do.

The earliest reference that we have found on this subject is An Analytical Model of Reliability Growth Through Testing by H. K. Weiss, Handbook No. 54 304, 17p, AD-035 767, May 1954, Northrop Aircraft Inc., Hawthrone, California. Also, a useful survey of some early reliability growth methods is Reliability Growth Modeling by Larry H. Crow, Technical Handbook No. 55, August 1972, U.S. Army Materiel Systems Analysis Agency, Aberdeen Proving Ground, Maryland.

The Duane Postulate.

James T. Duane, an engineer with General Electric's Motor and Generator Department, published a paper titled "Learning Curve Approach to Reliability Monitoring" in IEEE Transactions on Aerospace, Vol. 2, No. 2, 1964. This paper recorded his observation that if changes to improve reliability (which are now termed fixes) are incorporated into the design of a system under development, then on a log-log plot, the graph of cumulative failure rate Vs cumulative test time is linear. This observation has become known as the "Duane Postulate." This empirically derived statement is the key to the most commonly accepted growth model in use today (see Section 6.1.4). A graph given in Duane's paper is shown in Figure 6.1. The straight lines are based on a least squares fit of the data. The negative slope of each line is defined to be the growth rate, α, for that line.

Duane's Growth Model.

On a Log-Log Plot, the graph of

Cumulative Failure Rate
Vs
Cumulative Test Time

is Linear

Let N (t) = the total Number of failures by time t. Then the average failure rate, also called Cumulative failure rate C(t), can be found by dividing N(t) by t.

$$C(t) = \frac{N(t)}{t}$$

Let δ be the y-intercept on a log-log plot of the straight line that Duane postulated. The slope-intercept formula for this line then becomes:

$$Log\ C(t) \quad = \quad \delta - \alpha\ \ Log\, t$$

where log denotes the natural (base e) logarithm (although any base could be used).

The Duane Postulate:

$$Log\ C\,(\,t\,) = \delta - \alpha\, Log\ t$$

Taking anti-logs

$$C(t) \quad = \quad \lambda\ t^{-\alpha}$$

where

$$\delta \quad = \quad \ln \lambda$$

Multiplying C(t) by t gives N(t), and multiplying $t^{-\alpha}$ by t adds 1 to the exponent, $t^{1-\alpha}$.

So

$$N(t) \quad = \quad \lambda\, t^{1-\alpha}$$

Taking the first derivative of the number of failures with respect to time gives the instantaneous failure rate, r(t), at time t.

$$r(t) \quad = \quad \frac{d\,N(t)}{d\,t} \quad = \quad \lambda\left(1-\alpha\right)t^{-\alpha}$$

Duane's model thus has two parameters, α and λ. The first, α, determines the shape of the growth curve. The second, λ, is the size parameter for the curve. With these two parameters, the cumulative number of failures N(t), the average failure rate C(t), and the instantaneous failure rate r(t) can be calculated for any time t within the test. Further, given α and λ, it is possible to solve for t, the amount of testing time it will take to achieve a specific reliability. This assumes that the factors affecting reliability growth remain unchanged across the development.

Drawbacks to Duane's Method.

Duane stated that α could be universally treated as being .5, as that seemed to be the modal value within his database. This has since been shown to be unrealistic. It does not allow for different test environments causing failures to be surfaced at different rates, and for different levels of engineering effort causing different rates of fix insertion.

The reliability values calculated using his method are treated as being deterministic. That is, there is no allowance for the variation that is typically observed about an estimated value, and there is no way of judging whether the observed value, which rarely matches the estimated value, is close enough. Further, there is no way to check whether the model is valid for the current test situation.

All Duane growth curves pass through the origin of the graph. That is, the item under test is imputed to have zero reliability at the start of test.

The Crow/AMSAA Growth Model.

Larry H. Crow while at the U.S. Army Materiel Systems Analysis Activity's Reliability and Maintainability Division published Reliability Analysis for Complex, Repairable Systems, Technical Handbook No. 138, December 1975, U.S. AMSAA, Aberdeen Proving Ground, Maryland. In this handbook, Dr. Crow explored the advantages of using a Nonhomogeneous Poisson Process with a Weibull intensity function to model several phenomena, including reliability growth. If system failure times follow the Duane Postulate, then they can be modeled as a Nonhomogeneous Poisson Process with Weibull intensity function. To make the transition from Duane's formulae to the Weibull intensity functional forms, β has to be substituted for $1-\alpha$. Thus the parameters in the Crow model are λ and β, where β determines the shape of the curve. The physical interpretation of β (called the growth parameter) is the ratio of the current (instantaneous) MTBF to average (cumulative) MTBF at time t.

This stochastic interpretation immediately brings the benefits of Statistics to the formulae that Duane had derived. That is, the parameters λ and β can be determined using maximum likelihood estimators (mle's) rather than β being assumed to be fixed. Further, hypothesis tests and confidence limits can be determined for the parameters, and Goodness-of-Fit tests can be performed on the model. This eliminates the first two drawbacks of Duane's model. We will discuss later how Crow handles the problem of imputed zero reliability at the start of test.

One should take note that even though the growth rate estimate $\hat{\alpha}$ can be calculated from Crow's growth parameter estimate, $\hat{\beta}$, and it is still interpreted as the estimate of the negative slope of a straight line on a Log-Log plot, Crow's estimates of λ and β are somewhat different from the ones derived using Duane's procedures. This follows from the fact that the estimation procedure is mle, not least squares, thus each model's parameters correspond to different straight lines, respectively.

2. RELIABILITY GROWTH PLANNING

2.1 System Level Planning.

2.1.1 Introduction. The material in Section 2.1 is from Reference [1].

A well thought out reliability growth plan can serve as a significant management tool in scoping out the required resources to enhance system reliability and demonstrate the system reliability requirement. The principal goal of the growth test is to enhance reliability by the iterative process of surfacing failure modes, analyzing them, implementing corrective actions (fixes), and testing the "improved" configuration to verify fixes and continue the growth process by surfacing remaining failure modes. If the growth test environment during engineering and manufacturing development (EMD) reasonably simulates the mission environment stresses then it may be feasible to use the growth test data to statistically demonstrate the technical, i.e., engineering, requirement (denoted by TR) for system reliability. Such use of the growth test data could eliminate the need to conduct a follow-on reliability demonstration test. The classical demonstration test requires that the system configuration be held constant throughout the test. This type of test is principally conducted to assess and demonstrate the reliability of the configuration under test.

Associated with the demonstration test are statistical consumer and producer risks. In our context, they are frequently termed the Government and contractor risks, respectively. In broad terms, the Government risk is the probability of accepting a system when the true technical reliability is below the TR and the contractor risk is the probability of rejecting a system when the true technical reliability is at least the contractor's target value (set above the TR). An extensive amount of test time may be required for the reliability demonstration test to suitably limit these statistical risks. Moreover, this allotted test time would be principally devoted to demonstrating the system TR associated with the configuration under test instead of to enhancing the system reliability through the reliability growth process of sequential configuration improvement. In today's austere budgetary environment, it is especially important to make maximum use of test resources. With proper planning, a reliability growth program can be an efficient procedure for demonstrating the system reliability requirement while reliability improvements are being achieved via the growth process.

2.1.2 Background. During a reliability growth test phase, the system configuration is changing due to the activity of surfacing failure modes, analyzing the modes, and implementing fixes to the surfaced modes. It is often reasonable to portray this reliability growth in an idealized manner, i.e., by a smooth rising curve that captures the overall pattern of growth. The curve relates a measure of system reliability, e.g., mean-time-between-failures (MTBF), to test duration (e.g., hours). The functional form used to express this relationship in MIL-HDBK-189 [2] is given by

$$M(t) = \left(M_I / (1-\alpha)\right) \left(t/t_I\right)^{\alpha} \qquad (1)$$

In this equation, M(t) typically denotes the MTBF achieved after t test hours. The exponent α is termed the growth rate and represents the slope of the assumed linear relationship between ln{M(t)} and ln(t), where In denotes the base e logarithm function. The parameters t_I, M_I may be thought of as defining the initial conditions. In particular, M_I may be interpreted as the MTBF associated with the initial configuration entering the reliability growth test. In this interpretation, t_I would be the planned cumulative test time until one or more fixes are incorporated. An alternate and more general interpretation of M_I and t_I would be to regard M_I as the anticipated average MTBF over an initial test period t_I.

In the above discussion, we have referred to M(t) as the MTBF and have measured test duration by time units, e.g., t hours. We will continue to refer to M(t) and test duration t in this fashion; however, more generally, M(t) may denote mean-miles-to-failure or mean-rounds-to-failure (for a large number of rounds). The corresponding measures of test duration would be test mileage or rounds expended, respectively.

As indicated in Section 2.1.1, we shall consider using the data generated during the reliability growth test phase to demonstrate the system reliability technical requirement (TR) at a specified confidence level γ**.** This section addresses the case where the data consists of individual failure times $0<t_1<t_2< \ldots <t_n \le T$ for n observed mission reliability failures during test time T, where Equation (1) is assumed to hold for $0<t\le T$. Since the MIL-HDBK-189 growth model governed by Equation (1) is being assumed in this section, we shall also require that the observed number of failures by test duration t, denoted by N(t), be a non-homogeneous Poisson process with intensity function $\rho(t) = \{M(t)\}^{-1}$.

The growth curve planning parameters α, t_I, M_I, and the test time T should be chosen to reasonably limit the consumer (Government) and producer (contractor) statistical risks referred to in Section 2.1.1. Prior to presenting the relationship between these risks and the parameters mentioned above, it is instructive to review the determination of these risks for a reliability demonstration test based on a constant configuration.

The parameters defining the reliability demonstration test consist of the test duration T_{DEM}, and the allowable number of failures c. Define the random variable F_{obs} to be the number of failures that occur during the test time T_{DEM}. Denote the observed value of F_{obs} by f_{obs}. Then the "acceptance" or "passing" criterion is simply $f_{obs} \le c$.

Let M denote the MTBF associated with the constant configuration under test. Then F_{obs} has the Poisson probability distribution given by

$$\text{Prob}\,(F_{obs} = i) = e^{-T_{Dem}/M} \frac{(T_{Dem}/M)^i}{i!} \qquad (2)$$

Thus the probability of acceptance, denoted by Prob(A; M, c, T_{DEM}), as a function of M, c, and T_{DEM} is given by

$$\text{Prob}(A;M,c,T_{Dem}) = \text{Prob}(F_{obs} \le c) = \sum_{i=0}^{c} \text{Prob}(F_{obs} = i)$$

$$= \sum_{i=0}^{c} e^{-T_{Dem}/M} \frac{(T_{Dem}/M)^i}{i!} \qquad (3)$$

To ensure "passing the demonstration test" is equivalent to demonstrating the TR at confidence level γ (e.g., γ = 0.80 or γ = 0.90), we must choose c such that

$$f_{obs} \le c \quad \Leftrightarrow \quad TR \le \ell_\gamma(f_{obs}) \qquad (4)$$

where TR>0 and $\ell_\gamma(f_{obs})$ denotes the value of the 100 γ percent lower confidence bound when f_{obs} failures occur in the demonstration test of length T_{DEM}. Note that $\ell_\gamma(f_{obs})$ is a lower confidence bound on the true (but unknown) MTBF of the configuration under test. It is well known (see Proposition 1 in Appendix C) that the following choice of c satisfies (4):

Choose c to be the largest non-negative integer k that satisfies the inequality

$$\sum_{i=0}^{k} e^{-T_{Dem}/TR} \frac{(T_{Dem}/TR)^i}{i!} \le 1-\gamma \qquad (5)$$

Note c is well-defined provided

$$\exp(-T_{Dem}/TR) \le 1-\gamma \qquad (6)$$

Throughout this section we shall assume (6) holds and that c is defined as above.

Recall that the operating characteristic (OC) curve associated with a reliability demonstration test is the graph of the probability of acceptance, i.e., Prob (A;M,c,T_{DEM}) given in Equation (3), as a function of the true but unknown constant MTBF M as depicted on Figure 1.

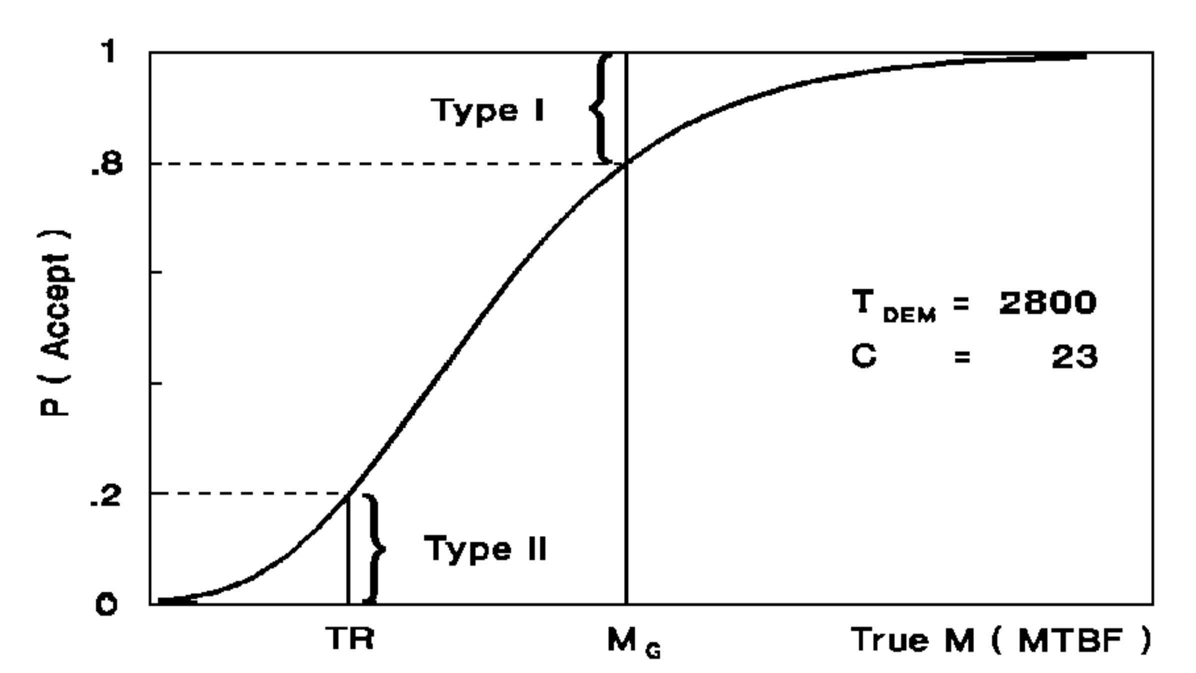

Figure 1. Example OC Curve for Reliability Demonstration Test.

The Government (or consumer risk) associated with this curve, called the Type II risk, is defined by

$$\text{Type II} \quad \triangleq \quad \text{Prob}\,(\,\text{A; TR, c, T}_{\text{Dem}}\,) \qquad (7)$$

Thus, by the choice of c,

$$\text{Type II} \quad \leq \quad 1 - \gamma \qquad (8)$$

For the contractor (producer) to have a reasonable chance of demonstrating the TR with confidence γ, the system configuration entering the reliability demonstration test must often have a MTBF value, say M_G (the contractor's goal MTBF) that is considerably higher than the TR. The probability that the producer fails the demonstration test given the system under test has a true MTBF value of M_G is termed the producer (contractor) or Type I risk. Thus

$$\text{Type I} \quad = \quad 1 \quad - \quad \text{Prob}\,(\,\text{A; M}_{\text{G}}\text{, c, T}_{\text{Dem}}\,) \qquad (9)$$

If the Type I risk is higher than desired, then either a higher value of M_G should be attained prior to entering the reliability demonstration test or T_{DEM} should be increased. If T_{DEM} is increased then c may have to be readjusted for the new value of T_{DEM} to remain the largest non-negative integer that satisfies inequality (5).

The above numbered equations and inequalities express the relationships between the reliability demonstration test parameters c, T_{DEM} , the requirement parameters TR, γ, and the associated risk parameters (the consumer and producer risks). These relationships are fundamental in conducting tradeoff analyses involving these parameters for planning reliability demonstration tests. In the next section we shall present relationships between the defining parameters for a reliability growth curve (M_I , t_I , α, and T), the requirement parameters (TR and γ), and the associated statistical risk parameters (the consumer and producer risks). Once these relationships are in hand, tradeoffs between these parameters may be utilized to consider demonstrating the TR at confidence level γ by utilizing reliability growth test data.

2.1.3 Reliability Growth Operating Characteristic (OC) Analysis. In the previous section, it was noted that for a reliability demonstration test, passing the test could be stated in terms of the allowable number of failures, c. It was noted that if c is properly chosen, then passing the test is equivalent to demonstrating the TR at confidence level γ, i.e.,

$$f_{obs} \leq c \quad \Leftrightarrow \quad TR \leq \ell\gamma\,(f_{obs})$$

In the presence of reliability growth, observing c or fewer failures is not equivalent to demonstrating the TR at a given confidence level. The cumulative times to failure as well as the number of failures must be considered when using reliability growth test data to demonstrate the TR at a specified confidence level γ. Thus, the "acceptance" or "passing" criterion must be stated directly in terms of the γ lower confidence bound on M(T) calculated from the reliability growth data. These data will be denoted by (n, s) where n is the number of failures occurring in the growth test of duration T and s = (t_1, t_2,…, t_n) is the vector of cumulative failure times. In particular, t_i denotes the cumulative test time to the i^{th} failure and 0<t_1 <t_2......<$t_n \leq T$ for $n \geq 1$. We shall also refer to the random vector (N, S) which takes on values (n, s) for $n \geq 1$. Unless otherwise stated, throughout the remainder of this report (N, S) will be conditioned on $N \geq 1$.

Using the lower confidence bound methodology developed for reliability growth data by Crow in [3], we shall define our acceptance criterion by the inequality

$$TR \leq \ell\gamma\ (n,\ s) \tag{10}$$

where $\ell_{\gamma}(n,s)$ is the γ statistical lower confidence bound on M(T), calculated as in [3] for $n \geq 1$. Thus, the probability of acceptance is given by

$$\mathrm{Prob}\ (TR \leq L_{\gamma}(N,S)) \tag{11}$$

where the random variable $L_{\gamma}(N,S)$ takes on the value $\ell_{\gamma}(n,s)$ when (N, S) takes on the value (n, s).

In accordance with [3], for $n \geq 1$, we define

$$\ell_{\gamma}(n,s) \;\underline{\Delta}\; \left(\frac{2n}{z_{\gamma}(n)}\right)^2 \hat{M}_n(T) \tag{12}$$

where $z_{\gamma}(n)$ is the unique positive value of z such that

$$(1/I_1(z)) \sum_{j=1}^{n} \frac{(z/2)^{2j-1}}{j!(j-1)!} = 1-\gamma \tag{13}$$

In the above, the function I_1 denotes the modified Bessel function of order one defined as follows:

$$I_1(z) \;\underline{\Delta}\; \sum_{j=1}^{\infty} \frac{(z/2)^{2j-1}}{j!(j-1)!} \tag{14}$$

In Equation (12), $\hat{M}_n(T)$ denotes the maximum likelihood estimate (mle) for M(T) given in MIL-HDBK-189 when n failures are observed. As discussed in MIL-HDBK-189,

$$\hat{M}_n(T) = T/\left(n\hat{\beta}_n\right) \tag{15}$$

where

$$\hat{\beta}_n = n/\left(\sum_{i=1}^{n} \ell n(T/t_i)\right) \tag{16}$$

The distribution of (N, S) and hence that of L_{γ} (N, S) is completely determined by the test duration T together with any set of parameters that define a unique reliability growth curve of the form given by Equation (1) in Section 2.1.2. Thus, the value of a probability expression such as given in (11) also depends on T and the assumed underlying growth curve parameters. One such set of parameters, as seen directly from Equation (1), is t_I, M_I, α together with T. In this growth curve representation, t_I may be arbitrarily chosen subject to 0<t_I<T. Alternately, scale parameter λ>0 and growth rate α, together with T, can be used to define the growth curve by the equation

$$M(t) = 1/(\lambda\beta t^{\beta-1}), \quad 0 < t \leq T \tag{17}$$

where $\beta = 1-\alpha$.

Note by Equation (17),

$$1/\lambda \quad = \quad (M(T))\beta T^{\beta-1} \tag{18}$$

Thus, the growth curve can also be expressed as

$$M(t) \quad = \quad (M(T))(t/T)^{\alpha}, \quad 0 < t \leq T \tag{19}$$

By Equation (19) we see that the distribution of (N, S) and hence that of L_γ (N, S) is determined by (α, T, M(T)).

Unless otherwise stated, throughout the remainder of this section, the distributions for (N, S) and for random variables defined in terms of (N, S) will be with respect to a fixed but unspecified set of values for α, T, M(T) subject only to $\alpha<1$, T>0, and M(T)>0. The same considerations apply to any associated probability expressions. In particular, the probability of acceptance, i.e., Prob (TR $\leq L_\gamma$(N, S)), is a function of (α, T, M(T)).

To further consider the probability of acceptance, we must first consider several properties of the system of lower confidence bounds generated by L_Υ (N, S) as specified via Equations (12) through (16). The statistical properties of this system of bounds directly follow from the properties of a set of conditional bounds derived by Crow in [3]. These latter bounds are conditioned on a sufficient statistic W that takes on the value

$$w \quad = \quad \sum_{i=1}^{n} \ell n\,(T/t_i) \tag{20}$$

when (N, S) takes on the value (n, s).

Let L_γ (N, S; w) denote the random variable L_γ (N, S) conditioned on W = w>0. In [3] Crow shows that L_γ (N, S; w) generates a system of γ lower confidence bounds on M(T), i.e.,

$$\text{Prob}\,(L_\gamma(N,S;w) \leq M(T)) \quad \geq \quad \gamma \tag{21}$$

for each set of values (α, T, M(T)) subject to $\alpha<1$, T>0, and M(T)>0. Note that the value of w is not known prior to conducting the reliability growth test. Thus, to calculate an OC curve for test planning, i.e., a priori, we wish to base our acceptance criterion on L_γ (N, S) as in (11) and not on the conditional random variable L_γ (N, S; w). We can utilize Equation (21) to show (see Propositions 2, 3, and 4 in Appendix C) that the Type II or consumer risk for M(T)=TR is at most 1-γ (for any $\alpha<1$ and T>0), analogous to the case in Section 2.1.2, i.e.,

$$\text{Type II} \quad = \quad \text{Prob}\,(TR \leq L_\gamma(N,S)) \quad \leq \quad 1-\gamma \tag{22}$$

for any $\alpha<1$ and T>0, provided M(T) = TR.

To emphasize the functional dependence of the probability of acceptance on the underlying true growth curve parameters (α, T, M(T)), we shall denote this probability by Prob (A; α, T, M(T)). Thus,

$$\text{Prob}\left(A;\alpha,T,M(T)\right) \;\underline{\Delta}\; \text{Prob}\left(TR \le L_{\gamma}(N,S)\right) \quad (23)$$

where the distribution of (N, S) and hence that of L_{γ} (N, S) is determined by (α,T, M(T)). It can be shown that Prob (A; α, T, M(T)) only depends on the values of M(T)/TR (or equivalently M(T) for known TR) and E(N). The ratio M(T)/TR is analogous to the discrimination ratio for a constant configuration reliability demonstration test of the type considered in Section 2.1.2. Note E(N) denotes the expected number of failures associated with the growth curve determined by (α, T, M(T)). More explicitly, the following equations can be derived (see Propositions 5 and 6 in Appendix C):

$$E(N) = T/\{(1-\alpha)M(T)\} \quad (24)$$

and

$$\text{Prob}\,(A;\alpha,\,T,\,M(T)) =$$

$$(1-e^{-\mu})^{-1}\sum_{n=1}^{\infty}\left[\text{Prob}\left(\frac{\chi_{2n}^{2}}{z_{\gamma}^{2}(n)} \ge \frac{1}{2\mu d}\right)\right] e^{-\mu}\left(\frac{\mu^{n}}{n!}\right) \quad (25)$$

where $\mu \;\underline{\Delta}\; E(N)$ and $d \;\underline{\Delta}\; M(T)/TR$.

Note (25) shows that the probability of acceptance only depends on μ and d. Thus, we shall subsequently denote the probability of acceptance by Prob (A;μ,d).

By (22),

$$\text{Type II} = \text{Prob}\left(A;\mu,1\right) \le 1-\gamma \quad (26)$$

Thus, the actual value of the Government or consumer risk solely depends on μ and is at most 1-γ. To consider the producer or contractor risk, Type I, let α_G denote the contractor's target or goal growth rate. This growth rate should be a value the contractor feels he can achieve for the growth test. Let M_G denote the contractor's MTBF goal. This is the MTBF value the contractor plans to achieve at the conclusion of the growth test of duration T. Thus, if the true growth curve has the parameters α_G and M_G, then the corresponding contractor risk of not demonstrating the TR at confidence level γ (utilizing the generated reliability growth test data) is given by

$$\text{Type I} = 1 - \text{Prob}(A; \mu_G, d_G) \tag{27}$$

where

$$d_G = M_G / TR \qquad \text{and} \qquad \mu_G = T / \{(1-\alpha_G) M_G\} \tag{28}$$

If the Type I risk is higher than desired, there are several ways to consider reducing this risk while maintaining the Type II risk at or below 1-γ. Since Prob (A; μ_G , d_G) is an increasing function of μ_G and d_G , the Type I risk can be reduced by increasing one or both of these quantities, e.g., by increasing T.

To further consider how the Type I statistical risk can be influenced, we shall express d_G and μ_G in terms of TR, T, α_G, and the initial conditions (M_I, t_I). Using Equations (1) and (19) with α = α_G and M(T) = M_G , by (28) we can show

$$M_G / TR = d_G = \left\{ \frac{M_I}{(1-\alpha_G)\, t_I^{\alpha_G}\, TR} \right\} T^{\alpha_G} \tag{29}$$

and

$$E(N) = \mu_G = (t_I^{\alpha_G} / M_I) T^{1-\alpha_G} \tag{30}$$

Note for a given requirement TR, initial conditions (M_I, t_I), and an assumed positive growth rate α_G, the contractor risk is a decreasing function of T via Equations (27), (29), and (30). These equations can be used to solve for a test time T such that the contractor risk is a specified value. The corresponding Government risk will be at most 1-γ and is given by Equation (26).

Section 2.1.4 contains two examples of an OC analysis for planning a reliability growth program. The first example illustrates the construction of an OC curve for given initial conditions (M_I, t_I) and requirement TR. The second example illustrates the iterative solution for the amount of test time T necessary to achieve a specified contractor (producer) risk, given initial conditions (M_I, t_I) and requirement TR. These examples use Equations (29) and (30) rewritten as in Equations (1) and (24), respectively, i.e.,

$$M(T) = \left(\frac{M_I}{1-\alpha} \right) \left(\frac{T}{t_I} \right)^{\alpha} \quad \text{and} \quad E(N) = \frac{T}{(1-\alpha) M(T)} \tag{31}$$

The quantities d= M(T)/TR and μ = E(N) are then used to obtain an approximation to Prob (A;μ,d). Approximate values are provided in Appendix B for a range of values for μ and d. The nature of this approximation is also discussed in Appendix B.

2.1.4 Application.

2.1.4.1 Example 1. Suppose we have a system under development that has a technical requirement (TR) MTBF of 100 hours to be demonstrated with 80 percent confidence. For the developmental program, a total of 2800 hours test time (T) at the system level has been predetermined for reliability growth purposes. Based on historical data for similar type systems and on lower level testing for the system under development, the initial MTBF (M_I) averaged over the first 500 hours (t_I) of system-level testing was expected to be 68 hours. Using these data, an idealized reliability growth curve was constructed such that if the tracking curve followed along the idealized growth curve, the TR MTBF of 100 hours would be demonstrated with 80 percent confidence. The growth rate (α) and the final MTBF (M(T)) for the idealized growth curve were 0.23 and 130 hours, respectively. The idealized growth curve for this program is depicted on Figure 2.

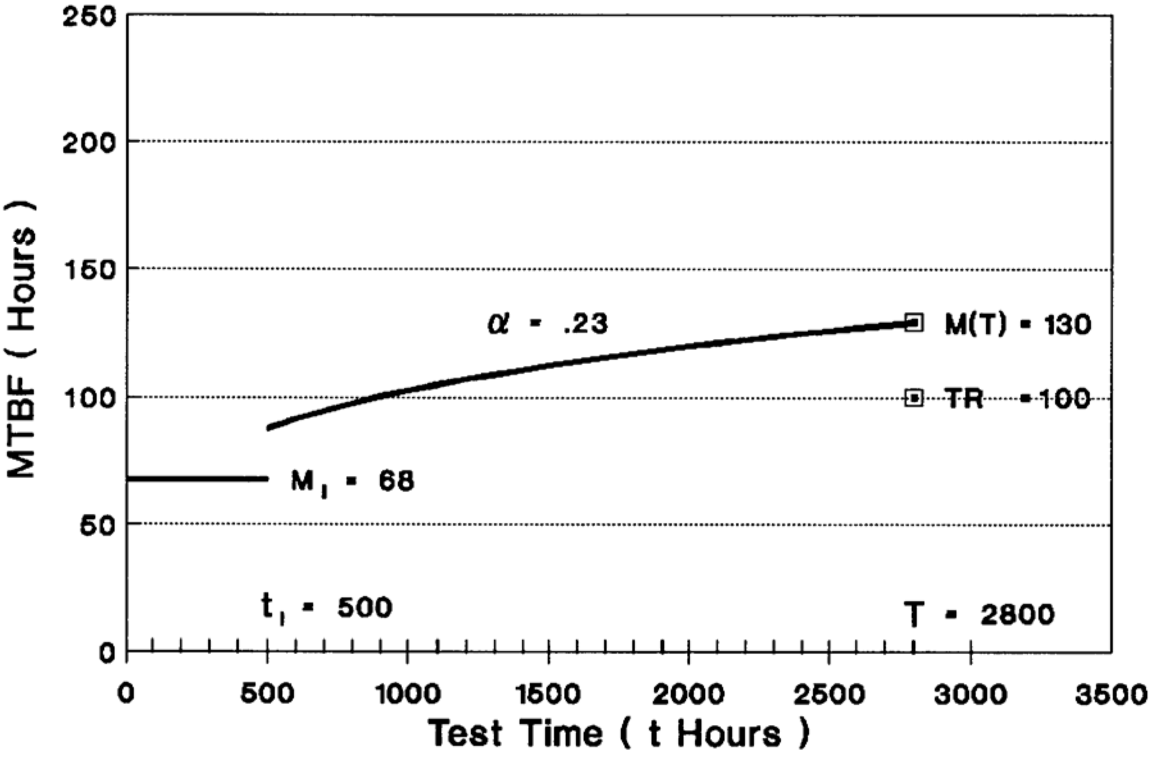

Figure 2. Idealized Reliability Growth Curve.

For this example, suppose we want to determine the operating characteristic (OC) curve for the program. For this, we need to consider alternate idealized growth curves where the M(T) vary but the M_I and t_I remain the same values as those for the program idealized growth curve; i.e., M_I = 68 hours and t_I = 500 hours. In varying the M(T), this is analogous to considering alternate values of the true MTBF for a reliability demonstration test of a fixed configuration system. For this program, one alternate idealized growth curve was determined where M(T) equals the TR whereas the remaining alternate idealized growth curves were determined for different values of the growth rate. These alternate idealized growth curves along with the program idealized growth curve are depicted on Figure 3.

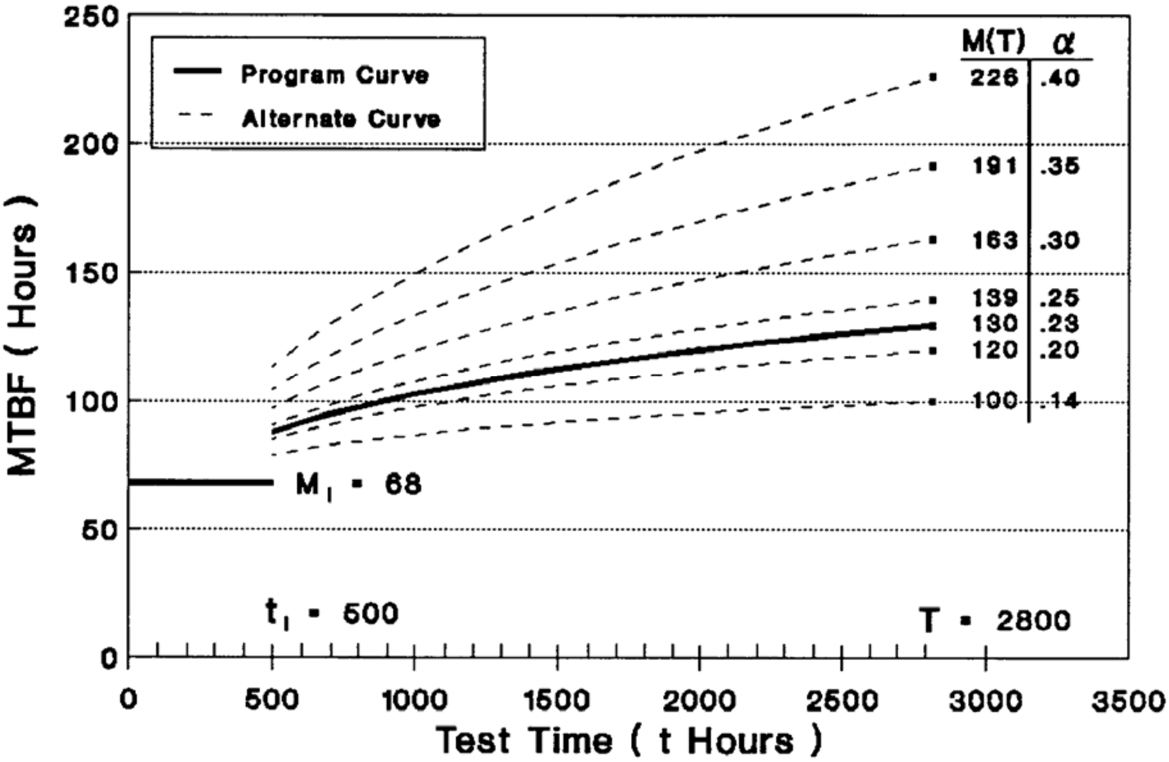

Figure 3. Program and Alternate Idealized Growth Curves.

Now, for each idealized growth curve we find M(T) and the expected number of failures E(N) from equation (31). Using the ratio M(T)/TR and E(N) as entries in the tables contained in Appendix B, we determine, by double linear interpolation, the probability of demonstrating the TR with 80 percent confidence. This probability is actually the probability that the 80 percent lower confidence bound (80 percent LCB) for M(T) will be greater than or equal to the TR. These probabilities represent the probability of acceptance (P(A)) points on the OC curve for this program which is depicted on Figure 4. The M(T), α, E(N), and P(A) for these idealized growth curves are summarized in the following table:

M (T)	α	E (N)	P (A)
100	0.14	32.6	0.15
120	0.20	29.2	0.37
130	0.23	28.0	0.48
139	0.25	26.9	0.58
163	0.30	24.5	0.77
191	0.35	22.6	0.90
226	0.40	20.6	0.96

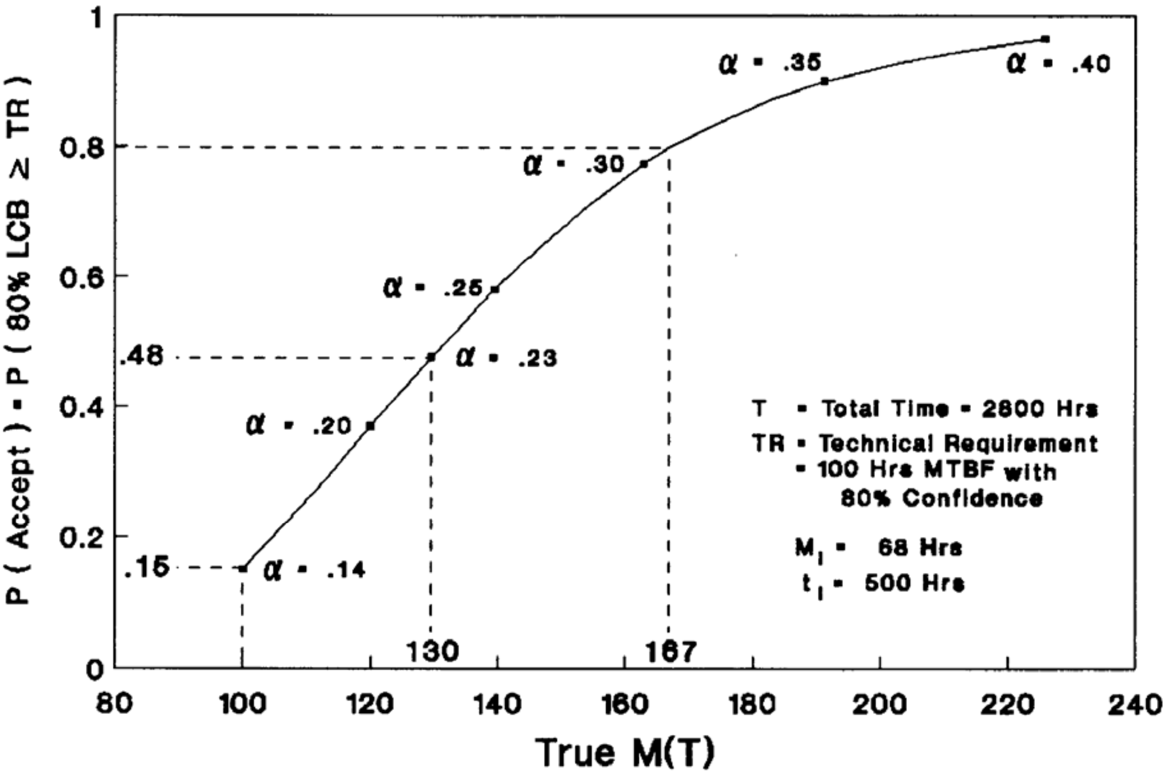

Figure 4. Operating Characteristic (OC) Curve.

From the OC curve, the Type I or producer risk is 0.52 (1-0.48) which is based on the program idealized growth curve where M(T) = 130. Note that if the true growth curve were the program idealized growth curve, there is still a 0.52 probability of not demonstrating the TR with 80 percent confidence. This occurs even though the true reliability would grow to M(T) = 130 which is considerably higher than the TR value of 100. The Type II or consumer risk, which is based on the alternate idealized growth curve where M(T) = TR = 100, is 0.15. As indicated on the OC curve, it should be noted that for this developmental program to have a producer risk of 0.20, the contractor would have to plan on an idealized growth curve with M(T) = 167.

2.1.4.2 Example 2. Consider a system under development that has a technical requirement (TR) MTBF of 100 hours to be demonstrated with 80 percent confidence, as in Example 1. The initial MTBF (M_I) over the first 500 hours (t_I) of system level testing for this system was estimated to be 48 hours which, again as in Example 1, was based on historical data for similar type systems and on lower level testing for the system under development. For this developmental program, it was assumed that a growth rate (α) of 0.30 would be appropriate for reliability growth purposes. Now, for this example, suppose we want to determine the total amount of system level test time (T) such that the Type I or producer risk for the program idealized reliability growth curve is 0.20; i.e., the probability of not demonstrating the TR of 100 hours with 80 percent confidence is 0.20 for the final MTBF value (M(T)) obtained from the program idealized growth curve. This probability corresponds to the probability of acceptance (P(A)) point of 0.80 (1-0.20) on the operating characteristic (OC) curve for this program.

Now, to determine the test time T which will satisfy the Type I or producer risk of 0.20, we first select an initial value of T and, as in Example 1, find M(T) and the expected number of failures (E(N)) from equation (31). Then, again, using the ratio M(T)/TR and E(N) as entries in the tables contained in Appendix B, we determine, by double linear interpolation, the probability of demonstrating the TR with 80 percent confidence. An iterative procedure is then applied until the P(A) obtained from the table equals the desired 0.80 within some reasonable accuracy. For this example, suppose we selected 3000 hours as our initial estimate of T and obtained the following iterative results:

T	M(T)	E(N)	P(A)
3000	117.4	36.5	<0.412
4000	128.0	44.6	<0.610
5000	136.8	52.2	<0.793
5500	140.8	55.8	0.815
5400	140.0	55.1	0.804
5300	139.2	54.4	0.790
5350	139.6	54.7	0.796
5375	139.8	54.9	0.800

Based on these results, we determine T = 5375 hours to be the required amount of system level test time such that the Type I or producer risk for the program idealized growth curve is 0.20.

2.1.5 Summary. The concepts of an operating characteristic (OC) analysis have been extended to the reliability growth setting. Government (consumer) and contractor (producer) statistical risks have been expressed in terms of the underlying growth curve parameters, test duration, and reliability requirement. In particular, for a given confidence level, these risks have been shown to depend solely on the expected number of failures during the growth test and the ratio of the MTBF to be achieved at the end of the growth program to the MTBF technical requirement to be demonstrated with confidence. Formulas have been developed for computing these risks as a function of the test duration and growth curve planning parameters.

The methodology developed and illustrated in this section should be of interest to RAM analysts responsible for structuring realistic reliability growth programs to achieve and demonstrate program objectives with reasonable statistical risks. In particular, this methodology allows the RAM analysts to construct a reliability growth curve that considers both the Government and contractor risks prior to agreeing to a reliability growth program.

2.2 Subsystem Level Planning.

2.2.1 Subsystem Reliability Growth. This material is based on Reference [4].

2.2.1.1 Benefits and Special Considerations. Conducting a subsystem reliability growth program prior to the start of system level testing can -

- reduce the amount of system level testing,
- reduce or eliminate many failure mechanisms (problem failure modes) early in the development cycle where they may be easier to locate and correct,
- allow for the use of subsystem test data to monitor reliability improvement,
- increase product quality by placing more emphasis on lower level testing and
- provide management with a strategy for conducting an overall reliability growth program.

Thus, subsystem reliability growth offers the potential for significant savings in testing cost.

To be an effective management tool for planning and assessing system reliability in the presence of reliability growth, it is important for the subsystem reliability growth process to adhere as closely as possible to the following considerations:

- Potential high-risk interfaces need to be identified and addressed through joint subsystem testing,
- Subsystem usage/test conditions need to be in conformance with the proposed system level operational environment as envisioned in the Operational Mode Summary/Mission Profile (OMS/MP),
- Failure Definitions/Scoring Criteria (FD/SC) formulated for each subsystem need to be consistent with the FD/SC used for system level test evaluation.

2.2.1.2 Overview of Subsystem Reliability Growth Planning Model – SSPLAN. The subsystem reliability growth planning model, SSPLAN, provides the user with a means to develop subsystem testing plans for demonstrating a system mean time between failures (MTBF) goal prior to system level testing. (The MTBF goal is also referred to as the MTBF objective ($MTBF_{obj}$).) In particular, the model is used to develop subsystem reliability growth planning curves that, with a specified probability, achieve a system MTBF objective with a specified confidence level. More precisely, associated with the subsystem MTBFs growing along a set of planned growth curves for given subsystem test durations is a probability; this is termed the probability of acceptance (P_A), the probability that the system MTBF objective will be met at the specified confidence level. The complement of P_A, 1-P_A, is termed the producer's (or contractor's) risk: the risk of not demonstrating the system MTBF objective at the specified confidence level when the subsystems are growing along their target growth curves for the prescribed test durations. Note that P_A also depends on the fixed MTBF of any non-growth subsystem and on the lengths of the demonstration tests on which the non-growth subsystem MTBF estimates are based.

SSPLAN estimates P_A for a given value of the final combined growth subsystem MTBF ($MTBF_{G,sys}$) by simulating the reliability growth of each subsystem and calculating a statistical lower confidence bound (LCB) for the final system MTBF based on the growth and non-growth subsystem simulated failure data. If the system LCB, at the specified confidence level, meets or exceeds the specified MTBF goal, then the trial is labeled a success. SSPLAN runs as many as 5000 trials, and estimates P_A as the number of successes divided by the number of trials.

One of the model's primary outputs is the growth subsystem test times required to meet the system level MTBF goal at the specified confidence level and P_A probability. The model determines the subsystem test times by using a specified fixed allocation of the combined final failure intensity to each of the individual growth subsystems.

As a reliability management tool, the model can serve as a means for prime contractors to coordinate/integrate the reliability growth activities of their subcontractors as part of their overall strategy in implementing a subsystem reliability test program for their developmental systems.

2.2.1.3 List of Notation. There are some variant terms in the following parameter list to show that the form of some parameters depends on the context in which they are used. For example, T, $T_{D,i}$ and $T_{G,i}$ indicate, respectively, that time may be used generically, specifically for non-growth subsystem i and specifically for growth subsystem i.

t	test time
T	total test time $(0 < t \leq T)$
F (t)	total number of subsystem failures by time t
E [F (t)]	expected number of subsystem failures by time t
λ	AMSAA model scale parameter $(\lambda > 0)$ for growth subsystem
β	AMSAA model shape (or growth) parameter $(\beta > 0)$ for growth subsystem
α	growth rate $(\alpha = 1-\beta)$, $(0<\alpha<1)$
t_I	initial time period for subsystem growth test $(t_I > 0)$
MTBF	Mean Time Between Failures
M_I	initial average MTBF over interval $(0, t_I]$, $(M_I > 0)$
λ_I	initial average failure intensity over interval $(0, t_I]$
ms	management strategy $(ms > 0)$
$\rho(T)$	instantaneous failure intensity at time T, $[\rho(T) > 0]$
M (T)	instantaneous MTBF at time T
$MTBF_{obj}$	system MTBF objective to be demonstrated with confidence γ

P_A	probability of acceptance associated with demonstrating $MTBF_{obj}$
LCB	lower confidence bound
D	demonstration (non-growth) test data or estimator
G	growth (test data or estimator)
i	subsystem index number
$T_{D,i}$	total amount of demonstration or "equivalent demonstration" (non-growth) test time for subsystem i
$T_{G,i}$	total amount of growth test time for subsystem i
$T_{MAX,i}$	specified maximum allowable growth test time for subsystem i. Thus $T_{G,i} \leq T_{MAX,i}$
$n_{D,i}$	number of failures during a demonstration test of length $T_{D,i}$ for a non-growth subsystem i. Also, number of "equivalent demonstration" failures for growth subsystem i during growth test
$n_{G,i}$	number of failures during a test time $T_{G,i}$ for a growth subsystem i
$M_{D,i}$	demonstration (constant) MTBF for non-growth subsystem i $\left(M_{D,i} > 0\right)$
$M_{G,i}$	Final MTBF for growth subsystem i
^	denotes an estimate when placed over a parameter
$\hat{\rho}_{D,i}\left(T_{D,i}\right)$	estimate of $M_{D,i}^{-1}$
$\rho_{G,i}(T_{G,i})$	equals $M_{G,i}^{-1}$
$\hat{\rho}_{G,i}(T_{G,i})$	estimate of $\rho_{G,i}(T_{G,i})$
χ_{df}^2	chi-squared random variable with "df" degrees of freedom
ρ_{SYS}	final system failure intensity
$\rho_{G,SYS}$	total failure intensity contribution of growth subsystems to ρ_{SYS}
a_i	fraction of $\rho_{G,SYS}$ allocated to growth subsystem i
$M_{G,SYS}$	final MTBF of combined growth subsystems, i.e., $M_{G,sys} = \rho_{G,sys}^{-1}$
$N_{D,SYS}$	system demonstration "equivalent" number of failures
$T_{D,SYS}$	system demonstration "equivalent" test time
mle	maximum likelihood estimate
~	symbol for "distributed as" a specified random variable
$\hat{M}_{D,i}$	subsystem i MTBF estimate of demonstration or "equivalent demonstration" MTBF
$\hat{M}_{G,i}$	subsystem i mle for final MTBF of growth subsystem
$\hat{M}_{SYS}$	estimate of final system MTBF
γ	specified confidence level for demonstrating $MTBF_{obj}$
$\chi_{df,\gamma}^2$	chi-squared 100γ percentile point for df degrees of freedom

$\hat{\rho}_i$ estimate of final subsystem i failure intensity
$\hat{\rho}_{SYS}$ estimate of final system failure intensity
K number of subsystems
$LCB_{D,i,\gamma}$ subsystem i LCB at γ confidence level from demonstration data
$LCB_{G,i,\gamma}$ subsystem i LCB at γ confidence level from growth data

The following terms are used for Cost calculations:
T_i amount of test time for subsystem i
$\left(C_F \right)_i$ cost per failure for subsystem i
$\left(C_T \right)_i$ cost per hour for subsystem i
C_{Total} total testing cost
$C_i[\Delta_{G,i}(T_{G,i})$ cost contribution of growth subsystem i to C_{total} as a function of $\Delta_{G,i}(T_{G,i})$
λ_i scale parameter for growth subsystem i
β_i shape parameter for growth subsystem i
α_i growth rate for growth subsystem i
$\left(M_{G,sys} \right)_{NEW}$ new value of $M_{G,\,sys}$ to use in search routine
$\left(M_{G,sys} \right)_{LB}$ lower bound for $M_{G,\,sys}$
$\left(M_{G,sys} \right)_{UB}$ upper bound for $M_{G,\,sys}$
$\left(P_A \right)_{LB}$ estimated P_A associated with $\left(M_{G,\,sys} \right)_{LB}$
$\left(P_A \right)_{UB}$ estimated P_A associated with $\left(M_{G,sys} \right)_{UB}$
$\left(P_A \right)_{GOAL}$ desired P_A

2.2.2 SSPLAN Methodology.

2.2.2.1 Model Assumptions. The SSPLAN methodology assumes that a system may be represented as a series of $K \geq 1$ independent subsystems. (The theory allows for $K = 1$ but the current computer implementation requires $K \geq 2$.)

$$\boxed{\text{System}} = \boxed{\text{Subsystem 1}} + \;\ldots\; + \boxed{\text{Subsystem K}}$$

This means that a failure of any single subsystem results in a system level failure and that a failure of a subsystem does not influence (either induce or prevent) the failure of any other subsystem. SSPLAN allows for a mixture of test data from growth and non-growth subsystems, but in its current implementation, at least one growth subsystem is required to run the model. The model utilizes the following assumption for the growth subsystems:

- The number of failures occurring over a period of test time follows a nonhomogeneous Poisson process (NHPP) with mean value function

$$E\left[F(t)\right] = \lambda t^{\beta} \qquad (\lambda,\beta,t>0) \tag{1}$$

where E[F(t)] is the expected number of failures by time t, λ is the scale parameter because it depends upon the unit of measurement chosen for t, and β is the growth (or shape) parameter because it characterizes the shape of the graph of the failure intensity function (the derivative of (1) with respect to t). The parameters λ and β may vary from subsystem to subsystem and will be subscripted by a subsystem index number when required for clarity. Non-growth subsystems are assumed to have constant failure rates.

2.2.2.2 Mathematical Basis for Growth Subsystems.

2.2.2.2.1 Initial Conditions. The power function shown in (1) together with the initial conditions described in this section provide a framework for a discussion of the way SSPLAN develops reliability growth curves. Together they provide a starting point for describing each growth subsystem's MTBF as a function of the parameters λ, β and t. Since λ is not convenient to directly work with for planning purposes, we shall relate λ to an initial or average subsystem MTBF over an initial period of test time. First, we note that the growth parameter, β, is related to the growth rate, α, by the following:

$$\beta = 1-\alpha \qquad (\beta>0) \tag{2}$$

For planned growth situations, α must be in the interval (0,1). Additional guidance on choosing α may be gained from Ellner & Trapnell [5].

The initial conditions for the model consist of:

- an initial time period, t_I, one choice for t_I is the amount of planned test items prior to the implementation of any corrective actions, and
- the initial reliability, M_I, representing the average reliability (MTBF) over the interval $\left(0,t_I\right]$.

From this, note that:

$$\lambda_I = \frac{1}{M_I} \qquad (M_I>0) \tag{3}$$

where λ_I is the average failure intensity over the interval $\left(0,t_I\right]$. The fact that (1) must be consistent with the initial conditions allows the scale parameter, λ, to be expressed in terms of planning parameters t_I, M_I, and α To do so, note the expected number of failures by time t_I is:

$$E\left[F(t_I)\right] = \lambda_I t_I \qquad (t_I>0) \tag{4}$$

Using (1), we see that the expected number of failures by time t_I is also given by

$$E\left[F\left(t_I\right)\right] \;=\; \lambda t_I^{\;\beta} \qquad \left(\lambda,\beta,t_I > 0\right) \tag{5}$$

By equating (4) and (5) and by using the relationship $\alpha = 1-\beta$ from (2), an expression for λ may be developed:

$$\lambda \;=\; \lambda_I\, t_I^{\;\alpha} = \frac{t_I^{\alpha}}{M_I} \qquad \left(t_I,\alpha > 0\right) \tag{6}$$

In addition to using both M_I and t_I as initial growth subsystem input parameters, the model allows a third possible input parameter, termed the planned management strategy ms, which represents the fraction of the initial subsystem failure intensity that is expected to be addressed through corrective actions. The relationships among these three parameters are revealed in the following discussion.

Since reliability growth occurs when correctable failure modes are surfaced and (successful) fixes are incorporated, it is desired to have a high probability of observing at least one correctable failure by time t_I. In what follows we shall utilize a probability of 0.95. From our assumptions, the number of failures that occur over the initial time period t_I, is Poisson distributed with expected value λ_I t_I. Thus

$$0.95 \;=\; 1-e^{-\left(ms\times\lambda_I\times t_I\right)} \;=\; 1-e^{-\left(\frac{ms\times t_I}{M_I}\right)} \quad (t_i, M_I > 0 \text{ and } 0 < ms \le 1)$$
(7)

From (7), it is evident that specifying any two of the parameters is sufficient in determining the third parameter. Thus, there are three options for the user when entering the initial conditions for growth subsystems.

2.2.2.2.2 Failure Intensity and Mean Time Between Failures – MTBF. The derivative with respect to time of the expected number of failures function (1) is:

$$\rho\left(t\right) \;=\; \lambda\,\beta\, t^{\beta-1} \qquad \left(\lambda,\beta,t > 0\right) \tag{8}$$

The function $\rho(t)$ represents the instantaneous failure intensity at time t. The reciprocal of $\rho(t)$ is the instantaneous MTBF at time t:

$$M\left(t\right) \;=\; \frac{1}{\rho\left(t\right)} \qquad \left(t,\rho\left(t\right) > 0\right) \tag{9}$$

Equations (8) and (9) provide much of the foundation for a discussion of how SSPLAN develops reliability growth curves for growth subsystems. Figure 5 shows a graphical representation of subsystem reliability growth.

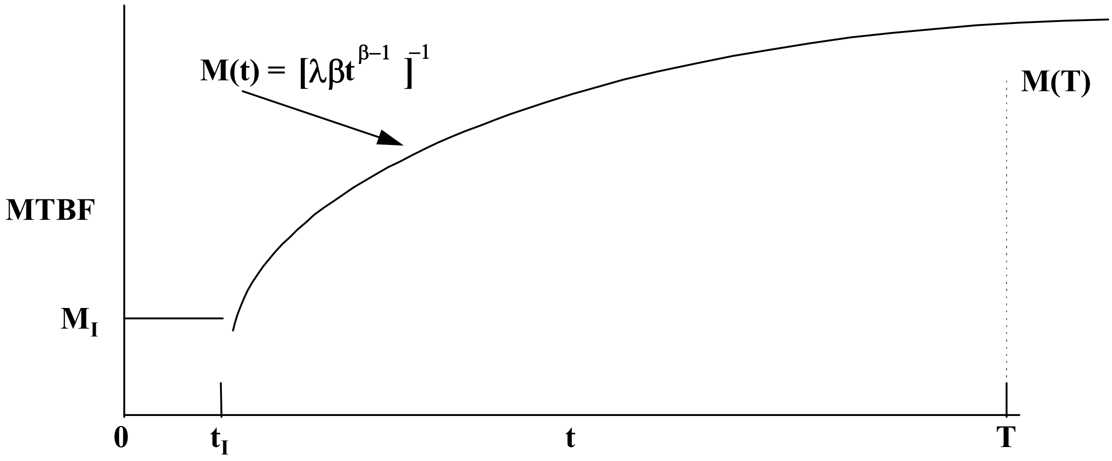

Figure 5. Reliability Growth Based On AMSAA Continuous Tracking Model.

2.2.2.3 Mathematical Basis for Non-growth Subsystems. Based on the constant failure rate assumption, the input parameters that characterize a non-growth subsystem are its fixed reliability estimate, M, and the length of the demonstration test, T, upon which the constant MTBF estimate is based.

2.2.2.4 Algorithm for Estimating Probability of Acceptance P_A. Rather than use purely analytical methods, SSPLAN uses simulation techniques to estimate the probability of achieving a system MTBF objective with a specified confidence level. This estimate of P_A is calculated by running the simulation a large number of trials.

Using the parameters that have been inputted and calculated at the subsystem level, the model generates "test data" for each subsystem for each simulation trial, thereby developing the data required to produce an estimate for the failure intensity for each subsystem. The test intervals and estimated failure intensities corresponding to the set of subsystems that comprise the system provide the necessary data for each trial of the simulation.

The model then uses a method developed for discrete data (the Lindström-Madden Method) to "roll up" the subsystem test data to arrive at an estimate for the final system reliability at a specified confidence level, namely, a statistical lower confidence bound (LCB) for the final system MTBF. In order for the Lindström-Madden method to be able to handle a mix of test data from both growth and non-growth subsystems, the model first converts all growth (G) subsystem test data to an "equivalent" amount of demonstration (D) test time and "equivalent" number of demonstration failures. This conversion process is done so that all subsystem results are expressed in a common format, namely, in terms of fixed configuration (non-growth) test data. (The equivalent demonstration test time and the equivalent demonstration number of failures are, respectively, the length of time and the number of failures a non-growth test would have to achieve to produce an {MTBF point estimate, MTBF LCB} pair that is equivalent to the respective estimates from a growth test.) By treating growth subsystem test data in this way, a standard lower confidence bound formula for time-truncated demonstration testing may be used to compute the system reliability LCB for the combination of "converted" growth and non-growth test data.

SSPLAN can run as many as 5000 trials. For each simulation trial, if the LCB for the final system MTBF meets or exceeds the specified system MTBF objective, then the trial is termed a success. An estimate for the probability of acceptance is the ratio of the number of successes to the number of trials.

The algorithm for estimating the probability of acceptance is described in greater detail by expanding upon the following four topics:

- generating "test data" estimates for growth subsystems
- generating "test data" estimates for non-growth subsystems
- converting growth subsystem data to "equivalent" demonstration data
- using the Lindström-Madden method for computing system level statistics

2.2.2.4.1 Generating Estimates for Growth Subsystems. There are two quantities of interest for each growth subsystem for each trial of the simulation -

- the total amount of test time, $T_{G,i}$, and
- the estimated failure intensity at that time, $\hat{\rho}_{G,i}\left(T_{G,i}\right)$.

To calculate $T_{G,i}$, note that from the initial input conditions we have values for the growth parameter, β (using (2)), and the scale parameter, λ (using (3) and (6)). Also, note that the final growth subsystem MTBF, $M_{G,i}$, can be calculated by dividing the final MTBF of the combined growth subsystems, $M_{G,SYS}$, by the subsystem failure intensity allocation. Equations (8) and (9) can then be combined and rearranged to solve for $T_{G,i}$:

$$T_{G,i} = \frac{1}{\left[\lambda \beta M_{G,i}\left(T_{G,i}\right)\right]^{\left[\frac{1}{(\beta-1)}\right]}} \qquad \left(\lambda, \beta, T_{G,i}, M_{G,i}\left(T_{G,i}\right) > 0; \beta \neq 1\right) \quad (10)$$

To generate the estimated failure intensity, $\hat{\rho}_{G,i}(T_{G,i})$, the model uses λ, β, $T_{G,i}$ and (1) with t = $T_{G,i}$ to calculate a Poisson distributed random number, $n_{G,i}$, which serves as an outcome for the number of growth failures during a simulation trial. The model then generates a chi-squared random number with $2n_{G,i}$ degrees of freedom and uses relation (12) below referenced in Crow [6] for obtaining a random value from the distribution for the estimated growth parameter, conditioned on the number of growth failures, $n_{G,i}$, during the trial:

$$\hat{\beta} \sim \frac{(2\beta n_{G,i})}{\chi^2_{2n_{G,i}}} \tag{12}$$

where β is obtained from the initial input and (2). One can show $n_{G,i}$, and the maximum likelihood estimates (mle's) for λ and β satisfy the following:

$$n_{G,i} = \hat{\lambda} T_{G,i}^{\hat{\beta}} \qquad \left(\hat{\lambda}, \hat{\beta}, T_{G,i} > 0\right) \tag{13}$$

In light of equation (1), this result is not surprising.

Using mle's for the parameters in (8) yields:

$$\hat{\rho}_{G,i}\left(T_{G,i}\right) = \hat{\lambda}\,\hat{\beta}\,T_{G,i}^{\hat{\beta}-1} \tag{14}$$

Rearranging terms in (14) we obtain:

$$\hat{\rho}_{G,i}\left(T_{G,i}\right) = \frac{\hat{\lambda}\,T_{G,i}^{\hat{\beta}}\,\hat{\beta}}{T_{G,i}} \tag{15}$$

Substituting (13) into (15) we conclude:

$$\hat{\rho}_{G,i}\left(T_{G,i}\right) = \frac{n_{G,i}\,\hat{\beta}}{T_{G,i}} \tag{16}$$

Thus using $n_{G,i}$, and the corresponding conditional estimate for β generated from (12), an estimate for the failure intensity, $\hat{\rho}_{G,i}(T_{G,i})$, can be obtained for each growth subsystem for each trial of the simulation.

2.2.2.4.2 Generating Estimates for Non-growth Subsystems. There are two quantities of interest for each non-growth subsystem for each trial of the simulation -

- the total amount of test time, $T_{D,i}$, and

- the estimated failure intensity, $\hat{\rho}_{D,i}(T_{D,i})$.

The total amount of test time, $T_{D,i}$, is an input planning parameter that represents the length of the demonstration test on which the non-growth subsystem MTBF estimate is based. To generate the estimated failure intensity, $\hat{\rho}_{D,i}(T_{D,i})$, the model first calculates (this is done only once for each non-growth subsystem in SSPLAN) the expected number of failures:

$$E\left[F\left(T_{D,i}\right)\right] = \frac{T_{D,i}}{M_{D,i}} \quad (18)$$

where $M_{D,i}$ is an input planning parameter representing the constant MTBF for the non-growth subsystem. The expected number of failures from (18) is then used as an input parameter (representing the mean of a Poisson distribution) to a routine that calculates a Poisson distributed random number, $n_{D,i}$, which is an outcome for the number of failures during a simulation trial. An estimate for the failure intensity follows:

$$\hat{\rho}_{D,i}\left(T_{D,i}\right) = \frac{n_{D,i}}{T_{D,i}} \quad (19)$$

2.2.2.4.3 Calculating Lower Confidence Bound for System MTBF. After all subsystem estimates have been calculated for a particular trial, SSPLAN uses a two-step approach to calculate the system reliability lower confidence bound by:

1. converting all growth subsystem data to "equivalent" demonstration data, that is, data from a fixed configuration. These data consist of:
   - $T_{D,i}$ - subsystem i equivalent demonstration test time and
   - $n_{D,i}$ - subsystem i equivalent demonstration number of failures

2. using the Lindström-Madden method to obtain system level statistics for calculating the LCB for the system MTBF.

2.2.2.4.3.1 Converting Growth Subsystem Data to "Equivalent" Demonstration Data. There are two equivalency relationships that must be maintained for the approach to be valid, namely, the demonstration data and the growth data must yield:

1. the same subsystem MTBF point estimate:

$$\hat{M}_{D,i} = \hat{M}_{G,i} \quad (20)$$

2. and the same subsystem MTBF lower bound at a specified confidence level γ:

$$LCB_{D,i,\gamma} = LCB_{G,i,\gamma} \quad (21)$$

Starting with the left side of the second equivalency relationship, (21), note that the lower confidence bound formula for time-truncated demonstration testing is:

$$LCB_{D,i,\gamma} \quad = \quad \frac{2\,T_{D,i}}{\chi^2_{2n_{D,i}+2,\gamma}} \tag{22}$$

where $T_{D,i}$ is the demonstration test time, $n_{D,i}$ is the demonstration number of failures, γ is the specified confidence level and $\chi^2_{2n_{D,i}+2,\gamma}$ is a chi-squared 100 γ percentile point with $2n_{D,i}+2$ degrees of freedom. Using an approximation equation developed by Crow, the lower confidence bound formula for growth testing (the right side of (21)) is:

$$LCB_{G,i,\gamma} \quad \approx \quad \frac{n_{G,i}\,\hat{M}_{G,i}}{\chi^2_{n_{G,i}+2,\gamma}} \tag{23}$$

where $n_{G,i}$ is the number of growth failures during the growth test, $\hat{M}_{G,i}$ is the mle for the MTBF and $\chi^2_{n_{G,i}+2,\gamma}$ is a chi-squared 100γ percentile point with $n_{G,i}+2$ degrees of freedom.

Since we want (22) and (23) to yield the same estimate, we begin by equating their denominators:

$$2n_{D,i}+2 = n_{G,i}+2 \quad \Rightarrow \quad n_{D,i} = \frac{n_{G,i}}{2} \tag{24}$$

Equating numerators from (22) and (23) yields:

$$2T_{D,i} = n_{G,i}\,\hat{M}_{G,i} \quad \Rightarrow \quad T_{D,i} = \frac{n_{G,i}\,\hat{M}_{G,i}}{2} \tag{25}$$

Dividing (25) by (24), and using (20) yields:

$$\hat{M}_{D,i} = \frac{T_{D,i}}{n_{D,i}} = \hat{M}_{G,i} \quad \Rightarrow \quad T_{D,i} = n_{D,i}\,\hat{M}_{G,i} \tag{26}$$

Substituting (24) into (26) yields:

$$T_{D,i} \quad = \quad \frac{n_{G,i}\,\hat{M}_{G,i}}{2} \tag{27}$$

Replacing the MTBF estimate by its failure intensity estimate from (14) yields:

$$T_{D,i} \quad = \quad \frac{n_{G,i}}{2\,\hat{\lambda}\,\hat{\beta}\,T_{G,i}^{\;\hat{\beta}-1}} \qquad \left(\hat{\lambda},\hat{\beta},T_{G,i} > 0\right) \qquad (28)$$

Multiplying both numerator and denominator of (28) by $T_{G,i}$, replacing the estimate of the expected number of failures (in the denominator) by the observed number of growth failures and canceling the term $n_{G,i}$ in the numerator and denominator yields:

$$T_{D,i} \quad = \quad \frac{T_{G,i}}{2\,\hat{\beta}} \qquad (29)$$

SSPLAN uses (24) and (29) in converting growth subsystem data to equivalent demonstration data.

2.2.2.4.3.2 Using the Lindström-Madden Method for Computing System Level Statistics. A continuous version of the Lindström-Madden method for discrete subsystems is used to compute an approximate lower confidence bound (LCB) for the final system MTBF from subsystem demonstration (non-growth) and "equivalent" demonstration (converted growth) data. The Lindström-Madden method typically generates a conservative LCB, which is to say the actual confidence level of the LCB is at least the specified level. It computes the following four estimates in order:

1. the equivalent amount of system level demonstration test time. (Since this estimate is the minimum demonstration test time of all the subsystems, it is constrained by the least tested subsystem.)

2. the estimate of the final system failure intensity, which is the sum of the estimated final growth subsystem failure intensities and non-growth subsystem failure rates

3. the "equivalent" number of system level demonstration failures, which is the product of the previous two estimates.

4. The approximate LCB for the final system MTBF at a given confidence level, which is a function of the equivalent amount of system level demonstration test time and the equivalent number of system level demonstration failures.

In equation form, these system level estimates are, respectively:

$$T_{D,Sys} \quad = \quad \min T_{D,i} \qquad \text{for i = 1..K} \qquad (30)$$

$$\hat{\rho}_{Sys} \quad = \quad \sum_{i=1}^{K} \hat{\rho}_i \qquad (31)$$

where $\hat{\rho}_i = \dfrac{1}{\hat{M}_{D,i}}$ and $\hat{M}_{D,i}$ = the MTBF estimate for subsystem i.

$$N_{D,Sys} = T_{D,Sys} \times \hat{\rho}_{Sys} \tag{32}$$

$$LCB_\gamma = \frac{2\,T_{D,Sys}}{\chi^2_{2N_{D,Sys}+2,\gamma}} \tag{33}$$

2.2.2.5 Calculation of Testing Costs. SSPLAN can be used to calculate the cost of carrying out a subsystem reliability growth plan for any given solution. The model does not address the initial start-up, or fixed costs since they are the same for any solution. The model does address all costs that are a function of the number of failures and all costs that are a function of time, as shown respectively in the following formula:

$$C_{Total} = \sum_{i \in \{all\,subsystems\}} \left\{ E\left[F_i(T_i)\right] \times (C_F)_i + T_i \times (C_T)_i \right\} \tag{34}$$

In (34), $E\left[F_i(T_i)\right]$ is the expected number of failures by time T_i for subsystem i, $\left(C_F\right)_i$ is the cost per failure for subsystem i, T_i is the amount of test time for subsystem i and $\left(C_T\right)_i$ is the cost per unit of time (usually per hour) for subsystem i. So, the total testing cost, C_{Total}, is the sum, over all subsystems, of the costs associated with testing each subsystem.

Once again, it is useful to treat growth and non-growth subsystems separately.

2.2.2.5.1 Calculating Cost for Growth Subsystems. For a given solution, we can calculate the cost contribution to C_{total} of a growth subsystem i in terms of $T_{G,i}$ and growth parameters λ_i, β_i by directly using (34) with $T_i = T_{G,i}$. Note by (1), $E\left[F_i\left(T_{G,i}\right)\right] = \lambda\, t_{G,i}^{\beta i}$. Alternately, we can express this cost in terms of the achieved subsystem failure intensity, $\Delta_{G,i}(T_{G,i})$, and λ_i, β_i, To write the cost equation in terms of the subsystem failure intensity, we begin by obtaining an expression for $T_{G,i}$ from (8):

$$\rho_{G,i}\left(T_{G,i}\right) = \lambda_i\, \beta_i\, T_{G,i}^{\beta_i - 1} \qquad \left(\lambda_i, \beta_i, T_{G,i} > 0\right) \tag{35}$$

Isolating the $T_{G,i}$ term on one side of (35) yields:

$$T_{G,i}^{\beta_i - 1} = \frac{\rho_{G,i}\left(T_{G,i}\right)}{\lambda_i\, \beta_i} \tag{36}$$

Raising both sides of (36) to the $1/\left(\beta_i - 1\right)$ power:

$$T_{G,i} = \frac{\left[\rho_{G,i}\left(T_{G,i}\right)\right]^{\left[\frac{1}{(\beta_i-1)}\right]}}{\left[\lambda_i \beta_i\right]^{\left[\frac{1}{(\beta_i-1)}\right]}} \qquad \left(\lambda_i,\beta_i,T_{G,i}\,\rho_{G,i} > 0; \beta_i \neq 1\right) \tag{37}$$

(Note $\beta_i \neq 1$ since subsystem i is a growth subsystem.)

Substituting from (2) yields the following intermediate result:

$$T_{G,i} = \left[\rho_{G,i}\left(T_{G,i}\right)\right]^{-\left(\frac{1}{\alpha_i}\right)}\left[\lambda_i\beta_i\right]^{\left(\frac{1}{\alpha_i}\right)} \qquad (0 < \alpha_i < 1) \tag{38}$$

Now, to obtain an expression for $E[F_i(T_i)]$, we begin with (1):

$$E\left[F_i\left(T_i\right)\right] = \lambda_i T_{G,i}^{\beta_i} \tag{39}$$

Substituting for $T_{G,i}$ from (38) yields:

$$E\left[F_i\left(T_i\right)\right] = \lambda_i\left[\rho_{G,i}\left(T_{G,i}\right)\right]^{-\left(\frac{\beta_i}{\alpha_i}\right)}\left[\lambda_i\beta_i\right]^{\left(\frac{\beta_i}{\alpha_i}\right)} \tag{40}$$

Rearranging terms in (40) yields:

$$E\left[F_i\left(T_i\right)\right] = \lambda_i^{\left(1+\frac{\beta_i}{\alpha_i}\right)}\left[\rho_{G,i}\left(T_{G,i}\right)\right]^{-\left(\frac{\beta_i}{\alpha_i}\right)}\beta_i^{\left(\frac{\beta_i}{\alpha_i}\right)} \tag{41}$$

Finally, the cost contribution in (34) of growth subsystem i can be expressed in terms of its failure intensity using (41) and (38):

$$\begin{aligned} C_i\left[\rho_{G,i}\left(T_{G,i}\right)\right] &= \left[\rho_{G,i}\left(T_{G,i}\right)\right]^{-\left(\frac{\beta_i}{\alpha_i}\right)}\lambda_i^{\left(1+\frac{\beta_i}{\alpha_i}\right)}\beta_i^{\left(\frac{\beta_i}{\alpha_i}\right)}\left(C_F\right)_i \\ &+\left[\rho_{G,i}\left(T_{G,i}\right)\right]^{-\left(\frac{1}{\alpha_i}\right)}\left[\lambda_i\beta_i\right]^{\left(\frac{1}{\alpha_i}\right)}\left(C_T\right)_i \end{aligned} \tag{42}$$

2.2.2.5.2 Calculating Cost for Non-growth Subsystems. To obtain the cost contribution of a non-growth subsystem, we use (18) to express $E[F_i(T_{D,i})]$ in terms of $T_{D,i}$ and $M_{D,i}$:

$$C_i\left[\rho_{D,i}\left(T_{D,i}\right)\right] = \left(\frac{T_{D,i}}{M_{D,i}}\right)\left(C_F\right)_i + T_{D,i}\left(C_T\right)_i \tag{43}$$

where $M_{D,i}^{-1} = \rho_{D,i}(T_{D,i})$

2.2.2.6 Methodology for a Fixed Allocation of Subsystem Failure Intensities. The methodology utilizes a fixed allocation, d_I, of $\Delta_{G,SYS}$ to each growth subsystem i thus $\Delta_{G,I} = d_i\Delta_{G,SYS}$. For this allocation, SSPLAN first determines if a solution exists that satisfies the criteria given by the user during the input phase. Specifically, SSPLAN checks to see if the desired probability of acceptance can be achieved with the given failure intensity allocations and maximum subsystem test times. If a solution does exist, SSPLAN will proceed to find the solution that meets the desired probability of acceptance within a small positive number epsilon.

2.2.2.6.1 Determining the Existence of a Solution. To determine if a solution is possible, SSPLAN uses (8) and (9) for each subsystem, with T set to the subsystem's maximum test time, to calculate the maximum possible MTBF for each subsystem. The maximum subsystem MTBF is multiplied by its failure intensity allocation to determine its influence on the system MTBF. For example, if a subsystem can grow to a maximum MTBF of 1000 hours and it has a failure intensity allocation of 0.5 (that is, its final failure intensity accounts for half of the total final failure intensity due to all of the growth subsystems), then that particular subsystem will limit the combined growth subsystem maximum MTBF to 500 hours. In other words, the maximum MTBF to which the growth portion of the system can grow, $MTBF_{G,sys}$, is the minimum of the products (subsystem final MTBF multiplied by the subsystem failure intensity allocation) from among all the growth subsystems.

The probability of acceptance, P_A, is then estimated using $MTBF_{G,sys}$. If the estimated P_A is less than the desired P_A, then no solution is possible within the limits of estimation precision for P_A, and SSPLAN will stop with a message to that effect.

2.2.2.6.2 Finding the Solution. On the other hand, if the estimated P_A is greater than or equal to the desired P_A, then a solution exists. If, by chance, the desired P_A has been met (within a small number epsilon) then SSPLAN will use $MTBF_{G,sys}$ as its solution. It is more likely, however, that the estimated P_A corresponding to $MTBF_{G,sys}$ exceeds the requirement, meaning that the program resulting in $MTBF_{G,sys}$ contains more testing than is necessary to achieve the desired P_A. SSPLAN proceeds, then, to find a value for $MTBF_{G,sys}$ that meets the desired P_A within epsilon.

To save time, P_A is initially estimated using a reduced number of iterations equal to one tenth of the requested number. As soon as the estimated P_A approaches the desired P_A, the full number of iterations is used.

For a given fixed failure intensity allocation, P_A increases as $MTBF_{G,sys}$ increases. Every value of $MTBF_{G,sys}$ determines a unique set of reliability growth curves, and thus a unique P_A. To find the set of growth curve test times that give rise to the desired P_A, SSPLAN first finds the upper and lower bounds for $MTBF_{G,sys}$. The initial upper bound for $MTBF_{G,sys}$ is the value found in verifying the existence of a solution; this value is the maximum possible value for $MTBF_{G,sys}$ (based on the maximum test times inputted by the user). The initial lower bound for $MTBF_{G,sys}$ is chosen arbitrarily; if the value chosen results in a P_A that is higher than the desired P_A, then the lower bound for $MTBF_{G,sys}$ is successively decreased until the resulting P_A is less than the desired P_A. At that point, upper and lower bounds for $MTBF_{G,sys}$ have been established, and SSPLAN uses a linear interpolation to find the value of $MTBF_{G,sys}$ that gives rise to an estimated P_A that meets the desired P_A. At each step of the search, $MTBF_{G,sys}$ is updated using the following equation (actually, the algorithm does all comparisons in terms of failure intensities, but the equation below shows the comparisons in terms of MTBFs to be consistent with Reference [4]):

$$\frac{\left(MTBF_{G,sys}\right)_{NEW} - \left(MTBF_{G,sys}\right)_{LB}}{\left(MTBF_{G,sys}\right)_{UB} - \left(MTBF_{G,sys}\right)_{LB}} = \frac{\left(P_A\right)_{GOAL} - \left(P_A\right)_{LB}}{\left(P_A\right)_{UB} - \left(P_A\right)_{LB}} \tag{44}$$

where $\left(MTBF_{G,sys}\right)_{UB}$ and $\left(MTBF_{G,sys}\right)_{LB}$ refer to the upper and lower bounds, respectively, for $MTBF_{G,sys}$; $\left(P_A\right)_{UB}$ and $\left(P_A\right)_{LB}$ refer to the estimated P_A values associated with each of the preceding $MTBF_{G,sys}$ values, respectively; and $\left(MTBF_{G,sys}\right)_{NEW}$ is the new value of $MTBF_{G,sys}$ to be used in the search algorithm.

The bounds are systematically updated during the search as follows. If the estimated value of P_A associated with $\left(MTBF_{G,sys}\right)_{NEW}$ is less than the desired probability of acceptance, $\left(P_A\right)_{GOAL}$, then $\left(MTBF_{G,sys}\right)_{NEW}$ becomes the new lower bound for the next search. If the estimated P_A is greater than the desired P_A, then $\left(MTBF_{G,sys}\right)_{NEW}$ becomes the new upper bound. The solution is found when the estimated P_A is within epsilon of the desired P_A or when the lower and upper bounds on $MTBF_{G,sys}$ are within epsilon of each other.

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APPENDIX B
TABLES

The following tables provide approximations to the probability of acceptance, $\Pr ob(A;\mu,d)$, where μ denotes the expected number of failures and d = M(T)/TR. The tabular entries were calculated using a modification to Equation (25). This modification entails (1) approximating $z_\gamma^2(\mathrm{n})$ by $4\mathrm{n}\chi^2_{\mathrm{n}+2,\gamma}$ and (2) conditioning on $N \geq 2$ instead of $N \geq 1$. Thus, in Equation (25) the expression $1 - \mathrm{e}^{-\mu}$ is replaced by $1 - \mathrm{P(N} \leq 1) = 1 - \mathrm{e}^{-\mu} - \mu\mathrm{e}^{-\mu}$ and the summation is over $N \geq 2$.

The approximation used for $z_\gamma^2(\mathrm{n})$ follows from the lower confidence bound approximation given by

$$\ell_\gamma(\mathrm{n,s}) \;\cong\; (\mathrm{n}/\chi^2_{\mathrm{n}+2,\gamma})\hat{M}_n \tag{32}$$

where $\hat{M}_n$ is the mle of M(T) calculated from the observed data s = (t_1, t_2,...t_n). Here t_i denotes the cumulative operating time to the i^{th} failure. This approximation was suggested by Dr. Larry Crow for conveniently approximating ℓ_γ(n, s). It has been our experience that the approximation in (32) results in slightly more conservative lower bounds on $M(T)$ than ℓ_γ(n, s). This implies that use of the corresponding approximation to $z_\gamma^2(\mathrm{n})$ would yield slightly smaller values of $\Pr ob(A;\mu,d)$ than one would obtain by utilizing $z_\gamma^2(\mathrm{n})$. Based on our experience with $\Pr ob(A;\mu,d)$ estimated by simulation, the approximating values appear to be within 0.01 of values obtained through simulation. We also observed that the approximation improves as n increases. The comparison between the lower confidence bound approximation given by (32) and the lower confidence bound using $z_\gamma^2(\mathrm{n})$ was based on Table C-2 contained in Section 3. Since the entries in this table were for $n \geq 2$, the probability of acceptance, $\Pr ob(A;\mu,d)$, was conditioned on $N \geq 2$. In most cases of interest for the model discussed in this report, Prob$(N \geq 2)$ will be close to one. In this situation, conditioning on $N \geq 2$ yields values of $\Pr ob(A;\mu,d)$ that are, for practical purposes, essentially the same as those obtained by conditioning on $N \geq 1$.

The entries in these tables were calculated using the well-known relationship between the complement of a Chi-square distribution function and the cumulative Poisson sum. This relationship was applied to calculate

$$\mathrm{Prob}\left(\chi^2_{2\mathrm{n}} \geq \frac{z_\gamma^2\,(\mathrm{n})}{2\mu\mathrm{d}}\right)$$

in the expression for $\Pr ob(A;\mu,d)$ in Section 2.1.3 with z_γ^2 (n) replaced by its approximation, i.e., $4\mathrm{n}\chi^2_{\mathrm{n}+2,\gamma}$. In terms of the cumulative Poisson sum, this yields

$$\text{Prob}\left(\chi^2_{2n} \geq \frac{2\,(n\chi^2_{n+2,\gamma})}{\mu d} \right) = \sum_{x=0}^{n-1} e^{-w} \frac{w^x}{x!}$$

(33)

where

$$w = (n\,\chi^2_{n+2,\gamma})/(\mu d).$$

With additional computational effort, one can more precisely calculate $\Pr ob(A;\mu,d)$ by iteratively solving for $z_\gamma(n)$ as the z-solution to Equation (13) of Section 2.1.3 over an appropriate range of n. Then Equation (33) can be utilized with $2n\chi^2_{n+2,\gamma}$ replaced by $z^2_\gamma(n)/2$.

The tables contained in this appendix are approximation values of $\Pr ob(A;\mu,d)$ for three confidence levels; namely, for $\gamma = 0.70$, $\gamma = 0.80$, and $\gamma = 0.90$.

TABLE FOR
70 PERCENT CONFIDENCE

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	5	6	7	8	9	10	11	12
1.00	0.131	0.150	0.163	0.173	0.180	0.186	0.191	0.195
1.05	0.150	0.171	0.187	0.199	0.208	0.216	0.224	0.230
1.10	0.169	0.194	0.212	0.226	0.238	0.249	0.258	0.266
1.15	0.189	0.217	0.238	0.255	0.269	0.282	0.294	0.304
1.20	0.209	0.240	0.264	0.284	0.301	0.316	0.330	0.343
1.25	0.231	0.264	0.291	0.314	0.334	0.351	0.368	0.383
1.30	0.252	0.289	0.319	0.344	0.367	0.387	0.405	0.422
1.35	0.274	0.314	0.347	0.375	0.400	0.422	0.443	0.462
1.40	0.296	0.339	0.375	0.405	0.432	0.457	0.479	0.500
1.45	0.318	0.364	0.402	0.435	0.465	0.491	0.515	0.538
1.50	0.340	0.389	0.430	0.465	0.496	0.525	0.550	0.574
1.55	0.362	0.414	0.457	0.494	0.527	0.557	0.584	0.609
1.60	0.384	0.438	0.484	0.523	0.557	0.588	0.616	0.642
1.65	0.406	0.462	0.510	0.550	0.586	0.618	0.647	0.673
1.70	0.427	0.486	0.535	0.577	0.614	0.647	0.676	0.703
1.75	0.448	0.509	0.560	0.603	0.641	0.674	0.704	0.730
1.80	0.469	0.531	0.583	0.628	0.666	0.700	0.729	0.756
1.85	0.489	0.553	0.606	0.651	0.690	0.724	0.754	0.780
1.90	0.509	0.575	0.628	0.674	0.713	0.746	0.776	0.802
1.95	0.529	0.595	0.650	0.695	0.734	0.768	0.797	0.822
2.00	0.548	0.615	0.670	0.716	0.754	0.787	0.816	0.840
2.05	0.566	0.634	0.689	0.735	0.773	0.806	0.833	0.857
2.10	0.584	0.652	0.708	0.753	0.791	0.823	0.849	0.872
2.15	0.601	0.670	0.725	0.770	0.807	0.838	0.864	0.885
2.20	0.618	0.687	0.742	0.786	0.823	0.853	0.877	0.898
2.25	0.634	0.703	0.758	0.802	0.837	0.866	0.890	0.909
2.30	0.650	0.719	0.773	0.816	0.850	0.878	0.901	0.919
2.35	0.665	0.733	0.787	0.829	0.863	0.889	0.911	0.928
2.40	0.679	0.747	0.800	0.841	0.874	0.900	0.920	0.936
2.45	0.693	0.761	0.813	0.853	0.884	0.909	0.928	0.943
2.50	0.706	0.774	0.825	0.864	0.894	0.917	0.936	0.950
2.55	0.719	0.786	0.836	0.874	0.903	0.925	0.942	0.955
2.60	0.732	0.797	0.846	0.883	0.911	0.932	0.948	0.960
2.65	0.743	0.808	0.856	0.892	0.918	0.938	0.954	0.965
2.70	0.755	0.818	0.865	0.900	0.925	0.944	0.958	0.969
2.75	0.766	0.828	0.874	0.907	0.932	0.950	0.963	0.973
2.80	0.776	0.837	0.882	0.914	0.937	0.954	0.967	0.976
2.85	0.786	0.846	0.889	0.920	0.943	0.959	0.970	0.978
2.90	0.795	0.855	0.896	0.926	0.947	0.963	0.973	0.981
2.95	0.804	0.862	0.903	0.932	0.952	0.966	0.976	0.983
3.00	0.813	0.870	0.909	0.937	0.956	0.969	0.979	0.985

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	13	14	15	16	17	18	19	20
1.00	0.199	0.202	0.206	0.208	0.211	0.213	0.215	0.217
1.05	0.235	0.241	0.245	0.250	0.254	0.258	0.262	0.265
1.10	0.274	0.281	0.288	0.294	0.300	0.306	0.311	0.316
1.15	0.314	0.323	0.332	0.340	0.348	0.355	0.363	0.369
1.20	0.355	0.366	0.377	0.387	0.397	0.406	0.415	0.423
1.25	0.397	0.410	0.423	0.435	0.446	0.457	0.467	0.477
1.30	0.438	0.454	0.468	0.481	0.494	0.507	0.519	0.530
1.35	0.480	0.496	0.512	0.527	0.542	0.555	0.568	0.581
1.40	0.520	0.538	0.555	0.572	0.587	0.602	0.616	0.629
1.45	0.559	0.578	0.597	0.614	0.630	0.646	0.660	0.674
1.50	0.596	0.617	0.636	0.654	0.671	0.687	0.702	0.716
1.55	0.632	0.653	0.673	0.691	0.709	0.725	0.740	0.754
1.60	0.666	0.687	0.707	0.726	0.743	0.759	0.774	0.788
1.65	0.697	0.719	0.739	0.758	0.775	0.791	0.805	0.819
1.70	0.727	0.749	0.769	0.787	0.804	0.819	0.833	0.846
1.75	0.754	0.776	0.796	0.813	0.829	0.844	0.857	0.869
1.80	0.780	0.801	0.820	0.837	0.852	0.866	0.879	0.890
1.85	0.803	0.823	0.842	0.858	0.873	0.886	0.897	0.908
1.90	0.824	0.844	0.861	0.877	0.891	0.903	0.913	0.923
1.95	0.843	0.862	0.879	0.894	0.906	0.918	0.927	0.936
2.00	0.861	0.879	0.895	0.908	0.920	0.930	0.939	0.947
2.05	0.877	0.894	0.908	0.921	0.932	0.941	0.949	0.956
2.10	0.891	0.907	0.921	0.932	0.942	0.951	0.958	0.964
2.15	0.903	0.919	0.931	0.942	0.951	0.959	0.965	0.970
2.20	0.915	0.929	0.941	0.950	0.959	0.965	0.971	0.976
2.25	0.925	0.938	0.949	0.958	0.965	0.971	0.976	0.980
2.30	0.934	0.946	0.956	0.964	0.970	0.976	0.980	0.984
2.35	0.942	0.953	0.962	0.969	0.975	0.980	0.984	0.987
2.40	0.949	0.959	0.967	0.974	0.979	0.983	0.987	0.989
2.45	0.955	0.965	0.972	0.978	0.982	0.986	0.989	0.991
2.50	0.961	0.969	0.976	0.981	0.985	0.989	0.991	0.993
2.55	0.966	0.973	0.979	0.984	0.988	0.990	0.993	0.994
2.60	0.970	0.977	0.982	0.987	0.990	0.992	0.994	0.995
2.65	0.974	0.980	0.985	0.989	0.991	0.993	0.995	0.996
2.70	0.977	0.983	0.987	0.990	0.993	0.995	0.996	0.997
2.75	0.980	0.985	0.989	0.992	0.994	0.996	0.997	0.998
2.80	0.982	0.987	0.991	0.993	0.995	0.996	0.997	0.998
2.85	0.984	0.989	0.992	0.994	0.996	0.997	0.998	0.998
2.90	0.986	0.990	0.993	0.995	0.996	0.997	0.998	0.999
2.95	0.988	0.992	0.994	0.996	0.997	0.998	0.999	0.999
3.00	0.990	0.993	0.995	0.996	0.998	0.998	0.999	0.999

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	21	22	23	24	25	26	27	28
1.00	0.219	0.221	0.223	0.224	0.225	0.227	0.228	0.229
1.05	0.269	0.272	0.275	0.278	0.280	0.283	0.286	0.288
1.10	0.321	0.326	0.331	0.335	0.339	0.343	0.347	0.351
1.15	0.376	0.382	0.388	0.394	0.400	0.406	0.411	0.417
1.20	0.432	0.440	0.447	0.455	0.462	0.469	0.476	0.482
1.25	0.487	0.496	0.505	0.514	0.523	0.531	0.539	0.547
1.30	0.541	0.552	0.562	0.572	0.581	0.590	0.599	0.608
1.35	0.593	0.605	0.616	0.626	0.637	0.646	0.656	0.665
1.40	0.642	0.654	0.666	0.677	0.688	0.698	0.708	0.717
1.45	0.688	0.700	0.712	0.723	0.734	0.745	0.755	0.764
1.50	0.729	0.742	0.754	0.765	0.776	0.786	0.796	0.805
1.55	0.767	0.780	0.792	0.803	0.813	0.823	0.832	0.841
1.60	0.801	0.813	0.825	0.835	0.845	0.854	0.863	0.871
1.65	0.831	0.843	0.854	0.863	0.873	0.881	0.889	0.897
1.70	0.858	0.868	0.878	0.888	0.896	0.904	0.911	0.918
1.75	0.881	0.891	0.900	0.908	0.916	0.923	0.929	0.935
1.80	0.900	0.910	0.918	0.925	0.932	0.939	0.944	0.949
1.85	0.917	0.926	0.933	0.940	0.946	0.951	0.956	0.961
1.90	0.931	0.939	0.946	0.952	0.957	0.962	0.966	0.970
1.95	0.944	0.950	0.956	0.961	0.966	0.970	0.973	0.977
2.00	0.954	0.960	0.965	0.969	0.973	0.977	0.980	0.982
2.05	0.962	0.967	0.972	0.976	0.979	0.982	0.984	0.986
2.10	0.969	0.974	0.977	0.981	0.984	0.986	0.988	0.990
2.15	0.975	0.979	0.982	0.985	0.987	0.989	0.991	0.992
2.20	0.980	0.983	0.986	0.988	0.990	0.992	0.993	0.994
2.25	0.984	0.986	0.989	0.991	0.992	0.994	0.995	0.996
2.30	0.987	0.989	0.991	0.993	0.994	0.995	0.996	0.997
2.35	0.989	0.991	0.993	0.994	0.995	0.996	0.997	0.998
2.40	0.991	0.993	0.995	0.996	0.996	0.997	0.998	0.998
2.45	0.993	0.995	0.996	0.997	0.997	0.998	0.998	0.999
2.50	0.995	0.996	0.997	0.997	0.998	0.998	0.999	0.999
2.55	0.996	0.997	0.997	0.998	0.998	0.999	0.999	0.999
2.60	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999
2.65	0.997	0.998	0.998	0.999	0.999	0.999	0.999	1.000
2.70	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000
2.75	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.80	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.85	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.90	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
2.95	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
3.00	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	29	30	31	32	33	34	35	36
1.00	0.231	0.232	0.233	0.234	0.235	0.236	0.236	0.237
1.05	0.291	0.293	0.295	0.297	0.300	0.302	0.304	0.306
1.10	0.355	0.359	0.362	0.366	0.369	0.373	0.376	0.379
1.15	0.422	0.427	0.432	0.437	0.441	0.446	0.450	0.455
1.20	0.489	0.495	0.501	0.507	0.513	0.519	0.525	0.530
1.25	0.554	0.562	0.569	0.576	0.582	0.589	0.596	0.602
1.30	0.616	0.624	0.632	0.640	0.648	0.655	0.662	0.669
1.35	0.674	0.683	0.691	0.699	0.707	0.715	0.722	0.729
1.40	0.727	0.735	0.744	0.752	0.760	0.767	0.775	0.782
1.45	0.773	0.782	0.790	0.798	0.806	0.813	0.820	0.827
1.50	0.814	0.822	0.830	0.838	0.845	0.852	0.859	0.865
1.55	0.849	0.857	0.864	0.871	0.878	0.884	0.890	0.896
1.60	0.879	0.886	0.893	0.899	0.905	0.911	0.916	0.921
1.65	0.904	0.910	0.916	0.922	0.927	0.932	0.936	0.941
1.70	0.924	0.930	0.935	0.940	0.944	0.948	0.952	0.956
1.75	0.941	0.946	0.950	0.954	0.958	0.961	0.965	0.968
1.80	0.954	0.958	0.962	0.965	0.969	0.971	0.974	0.976
1.85	0.965	0.968	0.971	0.974	0.977	0.979	0.981	0.983
1.90	0.973	0.976	0.978	0.981	0.983	0.985	0.986	0.988
1.95	0.979	0.982	0.984	0.986	0.987	0.989	0.990	0.991
2.00	0.984	0.986	0.988	0.990	0.991	0.992	0.993	0.994
2.05	0.988	0.990	0.991	0.992	0.993	0.994	0.995	0.996
2.10	0.991	0.992	0.994	0.994	0.995	0.996	0.997	0.997
2.15	0.993	0.994	0.995	0.996	0.997	0.997	0.998	0.998
2.20	0.995	0.996	0.997	0.997	0.998	0.998	0.998	0.999
2.25	0.996	0.997	0.998	0.998	0.998	0.999	0.999	0.999
2.30	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999
2.35	0.998	0.998	0.999	0.999	0.999	0.999	0.999	1.000
2.40	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.45	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.50	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
2.55	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	37	38	39	40	41	42	43	44
1.00	0.238	0.239	0.240	0.240	0.241	0.242	0.242	0.243
1.05	0.308	0.309	0.311	0.313	0.315	0.317	0.318	0.320
1.10	0.382	0.385	0.388	0.391	0.394	0.397	0.400	0.403
1.15	0.459	0.464	0.468	0.472	0.476	0.480	0.484	0.488
1.20	0.535	0.541	0.546	0.551	0.556	0.561	0.566	0.570
1.25	0.608	0.614	0.620	0.626	0.632	0.637	0.643	0.648
1.30	0.676	0.682	0.688	0.695	0.701	0.707	0.712	0.718
1.35	0.736	0.743	0.749	0.755	0.762	0.768	0.773	0.779
1.40	0.789	0.795	0.802	0.808	0.814	0.819	0.825	0.830
1.45	0.834	0.840	0.846	0.851	0.857	0.862	0.867	0.872
1.50	0.871	0.877	0.882	0.887	0.892	0.897	0.901	0.906
1.55	0.901	0.906	0.911	0.916	0.920	0.924	0.928	0.931
1.60	0.925	0.930	0.934	0.938	0.941	0.945	0.948	0.951
1.65	0.944	0.948	0.952	0.955	0.958	0.961	0.963	0.966
1.70	0.959	0.962	0.965	0.968	0.970	0.972	0.974	0.976
1.75	0.970	0.973	0.975	0.977	0.979	0.981	0.982	0.984
1.80	0.979	0.980	0.982	0.984	0.985	0.987	0.988	0.989
1.85	0.985	0.986	0.988	0.989	0.990	0.991	0.992	0.993
1.90	0.989	0.990	0.991	0.992	0.993	0.994	0.995	0.995
1.95	0.992	0.993	0.994	0.995	0.995	0.996	0.996	0.997
2.00	0.995	0.995	0.996	0.996	0.997	0.997	0.998	0.998
2.05	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999
2.10	0.997	0.998	0.998	0.998	0.999	0.999	0.999	0.999
2.15	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2.20	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.25	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.30	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	45	46	47	48	49	50	51	52
1.00	0.244	0.244	0.245	0.245	0.246	0.246	0.247	0.247
1.05	0.322	0.323	0.325	0.326	0.328	0.329	0.331	0.332
1.10	0.406	0.408	0.411	0.414	0.416	0.419	0.421	0.424
1.15	0.491	0.495	0.499	0.502	0.506	0.510	0.513	0.516
1.20	0.575	0.580	0.584	0.589	0.593	0.597	0.602	0.606
1.25	0.653	0.659	0.664	0.669	0.673	0.678	0.683	0.687
1.30	0.724	0.729	0.734	0.739	0.744	0.749	0.754	0.759
1.35	0.784	0.790	0.795	0.800	0.805	0.810	0.814	0.819
1.40	0.835	0.841	0.845	0.850	0.855	0.859	0.863	0.867
1.45	0.877	0.881	0.886	0.890	0.894	0.898	0.902	0.905
1.50	0.910	0.914	0.917	0.921	0.924	0.928	0.931	0.934
1.55	0.935	0.938	0.941	0.944	0.947	0.950	0.952	0.955
1.60	0.954	0.957	0.959	0.961	0.964	0.966	0.968	0.970
1.65	0.968	0.970	0.972	0.974	0.975	0.977	0.979	0.980
1.70	0.978	0.980	0.981	0.982	0.984	0.985	0.986	0.987
1.75	0.985	0.986	0.987	0.988	0.989	0.990	0.991	0.992
1.80	0.990	0.991	0.992	0.992	0.993	0.994	0.994	0.995
1.85	0.993	0.994	0.995	0.995	0.996	0.996	0.996	0.997
1.90	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.998
1.95	0.997	0.998	0.998	0.998	0.998	0.998	0.999	0.999
2.00	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
2.05	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.10	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	53	54	55	56	57	58	59	60
1.00	0.248	0.248	0.249	0.249	0.250	0.250	0.250	0.251
1.05	0.334	0.335	0.336	0.338	0.339	0.340	0.342	0.343
1.10	0.426	0.428	0.431	0.433	0.435	0.438	0.440	0.442
1.15	0.520	0.523	0.526	0.530	0.533	0.536	0.539	0.542
1.20	0.610	0.614	0.618	0.622	0.626	0.629	0.633	0.637
1.25	0.692	0.696	0.701	0.705	0.709	0.713	0.717	0.721
1.30	0.764	0.768	0.772	0.777	0.781	0.785	0.789	0.793
1.35	0.823	0.828	0.832	0.836	0.840	0.844	0.847	0.851
1.40	0.871	0.875	0.879	0.883	0.886	0.889	0.893	0.896
1.45	0.909	0.912	0.915	0.918	0.921	0.924	0.927	0.929
1.50	0.937	0.939	0.942	0.944	0.947	0.949	0.951	0.953
1.55	0.957	0.959	0.961	0.963	0.965	0.967	0.968	0.970
1.60	0.971	0.973	0.975	0.976	0.977	0.979	0.980	0.981
1.65	0.981	0.983	0.984	0.985	0.986	0.987	0.988	0.988
1.70	0.988	0.989	0.990	0.990	0.991	0.992	0.992	0.993
1.75	0.992	0.993	0.994	0.994	0.995	0.995	0.995	0.996
1.80	0.995	0.996	0.996	0.996	0.997	0.997	0.997	0.998
1.85	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.999
1.90	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
1.95	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.00	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	61	62	63	64	65	66	67	68
1.00	0.251	0.252	0.252	0.252	0.253	0.253	0.253	0.254
1.05	0.344	0.346	0.347	0.348	0.349	0.350	0.352	0.353
1.10	0.445	0.447	0.449	0.451	0.453	0.455	0.457	0.459
1.15	0.545	0.548	0.551	0.554	0.557	0.560	0.563	0.566
1.20	0.640	0.644	0.648	0.651	0.655	0.658	0.661	0.665
1.25	0.725	0.729	0.733	0.737	0.740	0.744	0.747	0.751
1.30	0.797	0.801	0.804	0.808	0.812	0.815	0.819	0.822
1.35	0.855	0.858	0.862	0.865	0.868	0.871	0.874	0.877
1.40	0.899	0.902	0.905	0.908	0.911	0.913	0.916	0.918
1.45	0.932	0.934	0.937	0.939	0.941	0.943	0.945	0.947
1.50	0.955	0.957	0.959	0.961	0.962	0.964	0.966	0.967
1.55	0.971	0.973	0.974	0.976	0.977	0.978	0.979	0.980
1.60	0.982	0.983	0.984	0.985	0.986	0.987	0.987	0.988
1.65	0.989	0.990	0.990	0.991	0.992	0.992	0.993	0.993
1.70	0.993	0.994	0.994	0.995	0.995	0.996	0.996	0.996
1.75	0.996	0.997	0.997	0.997	0.997	0.998	0.998	0.998
1.80	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
1.85	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.90	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	69	70	71	72	73	74	75	76
1.00	0.254	0.254	0.255	0.255	0.255	0.256	0.256	0.256
1.05	0.354	0.355	0.356	0.357	0.358	0.359	0.361	0.362
1.10	0.461	0.463	0.465	0.467	0.469	0.471	0.473	0.475
1.15	0.569	0.571	0.574	0.577	0.579	0.582	0.585	0.587
1.20	0.668	0.671	0.674	0.678	0.681	0.684	0.687	0.690
1.25	0.754	0.758	0.761	0.764	0.768	0.771	0.774	0.777
1.30	0.825	0.828	0.832	0.835	0.838	0.841	0.844	0.846
1.35	0.880	0.883	0.886	0.888	0.891	0.894	0.896	0.899
1.40	0.921	0.923	0.925	0.927	0.930	0.932	0.934	0.936
1.45	0.949	0.951	0.953	0.954	0.956	0.958	0.959	0.961
1.50	0.968	0.970	0.971	0.972	0.974	0.975	0.976	0.977
1.55	0.981	0.982	0.983	0.984	0.985	0.985	0.986	0.987
1.60	0.989	0.990	0.990	0.991	0.991	0.992	0.992	0.993
1.65	0.994	0.994	0.994	0.995	0.995	0.995	0.996	0.996
1.70	0.996	0.997	0.997	0.997	0.997	0.998	0.998	0.998
1.75	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999
1.80	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.85	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	77	78	79	80	81	82	83	84
1.00	0.256	0.257	0.257	0.257	0.257	0.258	0.258	0.258
1.05	0.363	0.364	0.365	0.366	0.367	0.368	0.369	0.370
1.10	0.477	0.479	0.481	0.483	0.485	0.486	0.488	0.490
1.15	0.590	0.593	0.595	0.598	0.600	0.602	0.605	0.607
1.20	0.693	0.696	0.699	0.702	0.704	0.707	0.710	0.713
1.25	0.780	0.783	0.786	0.789	0.792	0.795	0.797	0.800
1.30	0.849	0.852	0.855	0.857	0.860	0.862	0.865	0.867
1.35	0.901	0.903	0.906	0.908	0.910	0.912	0.914	0.916
1.40	0.937	0.939	0.941	0.943	0.944	0.946	0.948	0.949
1.45	0.962	0.963	0.965	0.966	0.967	0.968	0.969	0.971
1.50	0.978	0.979	0.980	0.980	0.981	0.982	0.983	0.984
1.55	0.987	0.988	0.989	0.989	0.990	0.990	0.991	0.991
1.60	0.993	0.993	0.994	0.994	0.994	0.995	0.995	0.995
1.65	0.996	0.997	0.997	0.997	0.997	0.997	0.998	0.998
1.70	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999
1.75	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.80	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	85	86	87	88	89	90	91	92
1.00	0.258	0.259	0.259	0.259	0.259	0.260	0.260	0.260
1.05	0.371	0.372	0.373	0.374	0.375	0.376	0.377	0.378
1.10	0.492	0.493	0.495	0.497	0.499	0.500	0.502	0.504
1.15	0.610	0.612	0.614	0.617	0.619	0.621	0.624	0.626
1.20	0.715	0.718	0.721	0.723	0.726	0.728	0.731	0.734
1.25	0.803	0.805	0.808	0.811	0.813	0.816	0.818	0.820
1.30	0.870	0.872	0.874	0.877	0.879	0.881	0.883	0.885
1.35	0.918	0.920	0.922	0.924	0.925	0.927	0.929	0.931
1.40	0.951	0.952	0.954	0.955	0.956	0.958	0.959	0.960
1.45	0.972	0.973	0.974	0.975	0.975	0.976	0.977	0.978
1.50	0.984	0.985	0.986	0.986	0.987	0.987	0.988	0.988
1.55	0.992	0.992	0.992	0.993	0.993	0.994	0.994	0.994
1.60	0.996	0.996	0.996	0.996	0.997	0.997	0.997	0.997
1.65	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999
1.70	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.75	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 70 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	93	94	95	96	97	98	99	100
1.00	0.260	0.260	0.261	0.261	0.261	0.261	0.261	0.262
1.05	0.379	0.379	0.380	0.381	0.382	0.383	0.384	0.385
1.10	0.506	0.507	0.509	0.510	0.512	0.514	0.515	0.517
1.15	0.628	0.630	0.633	0.635	0.637	0.639	0.641	0.643
1.20	0.736	0.738	0.741	0.743	0.746	0.748	0.750	0.753
1.25	0.823	0.825	0.827	0.830	0.832	0.834	0.836	0.839
1.30	0.887	0.889	0.891	0.893	0.895	0.897	0.899	0.901
1.35	0.932	0.934	0.935	0.937	0.938	0.940	0.941	0.942
1.40	0.961	0.962	0.963	0.964	0.965	0.966	0.967	0.968
1.45	0.979	0.979	0.980	0.981	0.982	0.982	0.983	0.984
1.50	0.989	0.989	0.990	0.990	0.991	0.991	0.991	0.992
1.55	0.994	0.995	0.995	0.995	0.995	0.996	0.996	0.996
1.60	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998
1.65	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.70	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
1.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE FOR
80 PERCENT CONFIDENCE

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	5	6	7	8	9	10	11	12
1.00	0.079	0.093	0.102	0.109	0.114	0.118	0.121	0.124
1.05	0.092	0.108	0.119	0.128	0.135	0.140	0.145	0.150
1.10	0.105	0.124	0.138	0.148	0.157	0.165	0.172	0.178
1.15	0.120	0.141	0.157	0.170	0.181	0.191	0.200	0.208
1.20	0.135	0.159	0.178	0.193	0.207	0.219	0.230	0.240
1.25	0.151	0.178	0.200	0.218	0.234	0.248	0.261	0.274
1.30	0.168	0.198	0.222	0.243	0.261	0.278	0.294	0.309
1.35	0.185	0.218	0.245	0.269	0.290	0.309	0.327	0.344
1.40	0.203	0.239	0.269	0.295	0.319	0.341	0.361	0.381
1.45	0.221	0.260	0.293	0.322	0.348	0.373	0.395	0.417
1.50	0.240	0.282	0.318	0.349	0.378	0.405	0.429	0.453
1.55	0.259	0.304	0.342	0.377	0.408	0.436	0.463	0.488
1.60	0.278	0.326	0.367	0.404	0.437	0.468	0.496	0.523
1.65	0.297	0.348	0.392	0.431	0.466	0.499	0.529	0.556
1.70	0.316	0.370	0.417	0.458	0.495	0.529	0.560	0.589
1.75	0.336	0.392	0.441	0.484	0.523	0.558	0.590	0.620
1.80	0.355	0.414	0.465	0.510	0.550	0.586	0.620	0.650
1.85	0.374	0.436	0.488	0.535	0.576	0.614	0.647	0.678
1.90	0.393	0.457	0.511	0.559	0.602	0.640	0.674	0.705
1.95	0.412	0.478	0.534	0.583	0.626	0.665	0.699	0.730
2.00	0.431	0.498	0.556	0.606	0.650	0.688	0.723	0.753
2.05	0.449	0.519	0.577	0.628	0.672	0.711	0.745	0.775
2.10	0.468	0.538	0.598	0.649	0.694	0.732	0.766	0.796
2.15	0.485	0.557	0.618	0.669	0.714	0.752	0.786	0.814
2.20	0.503	0.576	0.637	0.689	0.733	0.771	0.804	0.832
2.25	0.520	0.594	0.656	0.708	0.751	0.789	0.821	0.848
2.30	0.537	0.612	0.674	0.725	0.769	0.805	0.836	0.862
2.35	0.553	0.629	0.691	0.742	0.785	0.821	0.851	0.876
2.40	0.569	0.645	0.707	0.758	0.800	0.835	0.864	0.888
2.45	0.585	0.661	0.723	0.773	0.814	0.848	0.876	0.899
2.50	0.600	0.676	0.738	0.787	0.828	0.861	0.887	0.909
2.55	0.614	0.691	0.752	0.801	0.840	0.872	0.898	0.918
2.60	0.629	0.705	0.765	0.814	0.852	0.883	0.907	0.926
2.65	0.642	0.718	0.778	0.826	0.863	0.892	0.916	0.934
2.70	0.656	0.731	0.791	0.837	0.873	0.901	0.923	0.941
2.75	0.669	0.744	0.802	0.847	0.882	0.910	0.931	0.947
2.80	0.681	0.756	0.813	0.857	0.891	0.917	0.937	0.952
2.85	0.693	0.767	0.824	0.867	0.899	0.924	0.943	0.957
2.90	0.705	0.778	0.834	0.875	0.907	0.931	0.948	0.962
2.95	0.716	0.789	0.843	0.884	0.914	0.936	0.953	0.966
3.00	0.727	0.799	0.852	0.891	0.920	0.942	0.958	0.969

PROBABILITY OF DEMONSTRATING TECHNICAL REQUTREMEN@-
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	13	14	15	16	17	18	19	20
1.00	0.127	0.129	0.131	0.133	0.135	0.136	0.138	0.139
1.05	0.154	0.158	0.161	0.164	0.168	0.171	0.173	0.176
1.10	0.184	0.189	0.194	0.199	0.204	0.208	0.213	0.217
1.15	0.216	0.223	0.230	0.237	0.243	0.250	0.255	0.261
1.20	0.250	0.260	0.268	0.277	0.285	0.293	0.301	0.308
1.25	0.286	0.297	0.308	0.319	0.329	0.339	0.348	0.357
1.30	0.323	0.336	0.349	0.362	0.374	0.386	0.397	0.408
1.35	0.361	0.376	0.391	0.406	0.419	0.433	0.446	0.458
1.40	0.399	0.416	0.433	0.449	0.465	0.479	0.494	0.508
1.45	0.437	0.456	0.475	0.492	0.509	0.525	0.541	0.556
1.50	0.475	0.496	0.516	0.534	0.553	0.570	0.586	0.602
1.55	0.512	0.534	0.555	0.575	0.594	0.612	0.630	0.646
1.60	0.548	0.571	0.593	0.614	0.634	0.653	0.670	0.687
1.65	0.583	0.607	0.630	0.651	0.671	0.690	0.708	0.725
1.70	0.616	0.641	0.664	0.686	0.706	0.725	0.743	0.760
1.75	0.648	0.673	0.697	0.719	0.739	0.757	0.775	0.791
1.80	0.678	0.703	0.727	0.749	0.769	0.787	0.804	0.819
1.85	0.706	0.732	0.755	0.776	0.796	0.813	0.830	0.844
1.90	0.733	0.758	0.781	0.802	0.820	0.837	0.853	0.867
1.95	0.758	0.782	0.805	0.825	0.843	0.859	0.873	0.886
2.00	0.781	0.805	0.826	0.845	0.863	0.878	0.891	0.903
2.05	0.802	0.825	0.846	0.864	0.880	0.895	0.907	0.918
2.10	0.821	0.844	0.864	0.881	0.896	0.909	0.921	0.931
2.15	0.839	0.861	0.880	0.896	0.910	0.922	0.933	0.942
2.20	0.856	0.876	0.894	0.909	0.922	0.933	0.943	0.951
2.25	0.871	0.890	0.907	0.921	0.933	0.943	0.952	0.959
2.30	0.884	0.903	0.918	0.931	0.942	0.952	0.959	0.966
2.35	0.896	0.914	0.928	0.940	0.951	0.959	0.966	0.972
2.40	0.907	0.924	0.937	0.948	0.958	0.965	0.971	0.977
2.45	0.917	0.933	0.945	0.955	0.964	0.971	0.976	0.981
2.50	0.926	0.941	0.952	0.961	0.969	0.975	0.980	0.984
2.55	0.935	0.948	0.958	0.967	0.974	0.979	0.983	0.987
2.60	0.942	0.954	0.964	0.971	0.977	0.982	0.986	0.989
2.65	0.948	0.960	0.968	0.975	0.981	0.985	0.988	0.991
2.70	0.954	0.964	0.973	0.979	0.984	0.987	0.990	0.993
2.75	0.959	0.969	0.976	0.982	0.986	0.989	0.992	0.994
2.80	0.964	0.973	0.979	0.984	0.988	0.991	0.993	0.995
2.85	0.968	0.976	0.982	0.987	0.990	0.993	0.994	0.996
2.90	0.972	0.979	0.984	0.988	0.991	0.994	0.995	0.997
2.95	0.975	0.982	0.986	0.990	0.993	0.995	0.996	0.997
3.00	0.978	0.984	0.988	0.992	0.994	0.996	0.997	0.998

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	21	22	23	24	25	26	27	28
1.00	0.141	0.142	0.143	0.144	0.145	0.146	0.147	0.148
1.05	0.178	0.181	0.183	0.185	0.187	0.190	0.192	0.193
1.10	0.221	0.225	0.228	0.232	0.235	0.239	0.242	0.245
1.15	0.267	0.272	0.277	0.282	0.287	0.292	0.297	0.302
1.20	0.316	0.323	0.330	0.336	0.343	0.349	0.355	0.361
1.25	0.366	0.375	0.384	0.392	0.400	0.408	0.416	0.423
1.30	0.418	0.429	0.439	0.448	0.458	0.467	0.476	0.485
1.35	0.470	0.482	0.493	0.504	0.515	0.525	0.535	0.545
1.40	0.521	0.534	0.546	0.558	0.570	0.582	0.593	0.603
1.45	0.570	0.584	0.598	0.610	0.623	0.635	0.646	0.658
1.50	0.617	0.632	0.646	0.659	0.672	0.684	0.696	0.708
1.55	0.662	0.677	0.691	0.704	0.717	0.730	0.741	0.753
1.60	0.703	0.718	0.732	0.746	0.758	0.770	0.782	0.793
1.65	0.741	0.755	0.769	0.783	0.795	0.807	0.818	0.828
1.70	0.775	0.789	0.803	0.816	0.828	0.839	0.849	0.859
1.75	0.806	0.820	0.833	0.845	0.856	0.866	0.876	0.885
1.80	0.834	0.847	0.859	0.870	0.880	0.890	0.899	0.907
1.85	0.858	0.870	0.882	0.892	0.901	0.910	0.918	0.925
1.90	0.879	0.891	0.901	0.911	0.919	0.927	0.934	0.940
1.95	0.898	0.909	0.918	0.926	0.934	0.941	0.947	0.953
2.00	0.914	0.924	0.932	0.940	0.947	0.953	0.958	0.963
2.05	0.928	0.937	0.944	0.951	0.957	0.962	0.967	0.971
2.10	0.940	0.948	0.954	0.960	0.965	0.970	0.974	0.977
2.15	0.950	0.957	0.963	0.968	0.972	0.976	0.979	0.982
2.20	0.958	0.964	0.970	0.974	0.978	0.981	0.984	0.986
2.25	0.966	0.971	0.975	0.979	0.982	0.985	0.987	0.989
2.30	0.972	0.976	0.980	0.983	0.986	0.988	0.990	0.992
2.35	0.977	0.981	0.984	0.987	0.989	0.991	0.992	0.994
2.40	0.981	0.984	0.987	0.989	0.991	0.993	0.994	0.995
2.45	0.984	0.987	0.990	0.992	0.993	0.994	0.996	0.996
2.50	0.987	0.990	0.992	0.993	0.995	0.996	0.997	0.997
2.55	0.989	0.992	0.993	0.995	0.996	0.997	0.997	0.998
2.60	0.991	0.993	0.995	0.996	0.997	0.997	0.998	0.998
2.65	0.993	0.995	0.996	0.997	0.997	0.998	0.998	0.999
2.70	0.994	0.996	0.997	0.997	0.998	0.998	0.999	0.999
2.75	0.995	0.996	0.997	0.998	0.998	0.999	0.999	0.999
2.80	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999
2.85	0.997	0.998	0.998	0.999	0.999	0.999	0.999	1.000
2.90	0.997	0.998	0.999	0.999	0.999	0.999	1.000	1.000
2.95	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000
3.00	0.998	0.999	0.999	0.999	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	29	30	31	32	33	34	35	36
1.00	0.149	0.149	0.150	0.151	0.151	0.152	0.153	0.153
1.05	0.195	0.197	0.199	0.201	0.202	0.204	0.206	0.207
1.10	0.248	0.251	0.254	0.257	0.260	0.263	0.266	0.269
1.15	0.306	0.311	0.315	0.319	0.324	0.328	0.332	0.336
1.20	0.367	0.373	0.379	0.385	0.390	0.396	0.401	0.407
1.25	0.430	0.438	0.445	0.452	0.459	0.465	0.472	0.478
1.30	0.494	0.502	0.510	0.518	0.526	0.534	0.542	0.549
1.35	0.555	0.564	0.574	0.582	0.591	0.600	0.608	0.616
1.40	0.614	0.624	0.633	0.643	0.652	0.661	0.670	0.678
1.45	0.668	0.679	0.689	0.699	0.708	0.717	0.726	0.735
1.50	0.718	0.729	0.739	0.749	0.758	0.767	0.776	0.784
1.55	0.764	0.774	0.784	0.793	0.802	0.811	0.819	0.827
1.60	0.803	0.813	0.823	0.832	0.840	0.848	0.856	0.863
1.65	0.838	0.847	0.856	0.864	0.872	0.879	0.886	0.893
1.70	0.868	0.876	0.884	0.892	0.899	0.905	0.911	0.917
1.75	0.893	0.901	0.908	0.915	0.921	0.926	0.932	0.937
1.80	0.914	0.921	0.927	0.933	0.938	0.943	0.948	0.952
1.85	0.932	0.938	0.943	0.948	0.953	0.957	0.961	0.964
1.90	0.946	0.951	0.956	0.960	0.964	0.967	0.971	0.973
1.95	0.957	0.962	0.966	0.969	0.973	0.976	0.978	0.980
2.00	0.967	0.971	0.974	0.977	0.979	0.982	0.984	0.986
2.05	0.974	0.977	0.980	0.983	0.985	0.987	0.988	0.990
2.10	0.980	0.983	0.985	0.987	0.989	0.990	0.991	0.993
2.15	0.985	0.987	0.989	0.990	0.992	0.993	0.994	0.995
2.20	0.988	0.990	0.991	0.993	0.994	0.995	0.996	0.996
2.25	0.991	0.992	0.994	0.995	0.995	0.996	0.997	0.997
2.30	0.993	0.994	0.995	0.996	0.997	0.997	0.998	0.998
2.35	0.995	0.996	0.997	0.997	0.998	0.998	0.998	0.999
2.40	0.996	0.997	0.997	0.998	0.998	0.999	0.999	0.999
2.45	0.997	0.998	0.998	0.998	0.999	0.999	0.999	0.999
2.50	0.998	0.998	0.999	0.999	0.999	0.999	0.999	1.000
2.55	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.60	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.65	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.70	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
2.75	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	37	38	39	40	41	42	43	44
1.00	0.154	0.154	0.155	0.156	0.156	0.157	0.157	0.157
1.05	0.209	0.210	0.212	0.213	0.214	0.216	0.217	0.218
1.10	0.271	0.274	0.276	0.279	0.281	0.284	0.286	0.289
1.15	0.340	0.344	0.347	0.351	0.355	0.359	0.362	0.366
1.20	0.412	0.417	0.422	0.427	0.432	0.437	0.441	0.446
1.25	0.485	0.491	0.497	0.503	0.509	0.515	0.521	0.526
1.30	0.556	0.563	0.570	0.577	0.584	0.591	0.597	0.604
1.35	0.624	0.632	0.639	0.647	0.654	0.661	0.668	0.675
1.40	0.687	0.695	0.703	0.710	0.718	0.725	0.732	0.739
1.45	0.743	0.751	0.759	0.766	0.773	0.781	0.787	0.794
1.50	0.792	0.800	0.807	0.814	0.821	0.828	0.834	0.841
1.55	0.834	0.842	0.848	0.855	0.861	0.867	0.873	0.879
1.60	0.870	0.876	0.883	0.888	0.894	0.899	0.905	0.909
1.65	0.899	0.905	0.910	0.915	0.920	0.925	0.929	0.933
1.70	0.923	0.928	0.932	0.937	0.941	0.945	0.948	0.952
1.75	0.941	0.946	0.949	0.953	0.957	0.960	0.963	0.965
1.80	0.956	0.960	0.963	0.966	0.969	0.971	0.973	0.976
1.85	0.967	0.970	0.973	0.975	0.977	0.979	0.981	0.983
1.90	0.976	0.978	0.980	0.982	0.984	0.986	0.987	0.988
1.95	0.982	0.984	0.986	0.987	0.989	0.990	0.991	0.992
2.00	0.987	0.989	0.990	0.991	0.992	0.993	0.994	0.995
2.05	0.991	0.992	0.993	0.994	0.995	0.995	0.996	0.996
2.10	0.994	0.994	0.995	0.996	0.996	0.997	0.997	0.998
2.15	0.995	0.996	0.997	0.997	0.997	0.998	0.998	0.998
2.20	0.997	0.997	0.998	0.998	0.998	0.999	0.999	0.999
2.25	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999
2.30	0.998	0.999	0.999	0.999	0.999	0.999	0.999	1.000
2.35	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.40	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.45	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	45	46	47	48	49	50	51	52
1.00	0.158	0.158	0.159	0.159	0.160	0.160	0.160	0.161
1.05	0.220	0.221	0.222	0.224	0.225	0.226	0.227	0.228
1.10	0.291	0.294	0.296	0.298	0.300	0.303	0.305	0.307
1.15	0.369	0.373	0.376	0.380	0.383	0.386	0.390	0.393
1.20	0.451	0.455	0.460	0.464	0.469	0.473	0.478	0.482
1.25	0.532	0.538	0.543	0.548	0.554	0.559	0.564	0.569
1.30	0.610	0.616	0.622	0.628	0.634	0.640	0.645	0.651
1.35	0.681	0.688	0.694	0.701	0.707	0.713	0.718	0.724
1.40	0.745	0.752	0.758	0.764	0.770	0.776	0.782	0.787
1.45	0.800	0.807	0.813	0.818	0.824	0.829	0.835	0.840
1.50	0.846	0.852	0.858	0.863	0.868	0.873	0.878	0.882
1.55	0.884	0.889	0.894	0.899	0.903	0.907	0.911	0.915
1.60	0.914	0.918	0.922	0.926	0.930	0.934	0.937	0.940
1.65	0.937	0.941	0.944	0.947	0.950	0.953	0.956	0.959
1.70	0.955	0.958	0.960	0.963	0.965	0.968	0.970	0.972
1.75	0.968	0.970	0.972	0.974	0.976	0.978	0.980	0.981
1.80	0.978	0.979	0.981	0.983	0.984	0.985	0.986	0.988
1.85	0.984	0.986	0.987	0.988	0.989	0.990	0.991	0.992
1.90	0.989	0.990	0.991	0.992	0.993	0.994	0.994	0.995
1.95	0.993	0.994	0.994	0.995	0.995	0.996	0.996	0.997
2.00	0.995	0.996	0.996	0.997	0.997	0.997	0.998	0.998
2.05	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999
2.10	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999
2.15	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000
2.20	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.25	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	53	54	55	56	57	58	59	60
1.00	0.161	0.161	0.162	0.162	0.162	0.163	0.163	0.163
1.05	0.230	0.231	0.232	0.233	0.234	0.235	0.236	0.237
1.10	0.309	0.311	0.313	0.316	0.318	0.320	0.322	0.324
1.15	0.396	0.399	0.403	0.406	0.409	0.412	0.415	0.418
1.20	0.486	0.490	0.494	0.498	0.502	0.506	0.510	0.514
1.25	0.574	0.579	0.584	0.589	0.593	0.598	0.603	0.607
1.30	0.656	0.662	0.667	0.672	0.677	0.682	0.687	0.692
1.35	0.730	0.735	0.740	0.746	0.751	0.756	0.761	0.766
1.40	0.793	0.798	0.803	0.808	0.813	0.818	0.822	0.827
1.45	0.845	0.850	0.854	0.859	0.863	0.867	0.872	0.876
1.50	0.887	0.891	0.895	0.899	0.902	0.906	0.910	0.913
1.55	0.919	0.922	0.926	0.929	0.932	0.935	0.938	0.941
1.60	0.943	0.946	0.949	0.951	0.954	0.956	0.958	0.961
1.65	0.961	0.963	0.965	0.967	0.969	0.971	0.973	0.974
1.70	0.974	0.975	0.977	0.979	0.980	0.981	0.982	0.984
1.75	0.983	0.984	0.985	0.986	0.987	0.988	0.989	0.990
1.80	0.989	0.990	0.990	0.991	0.992	0.993	0.993	0.994
1.85	0.993	0.993	0.994	0.994	0.995	0.995	0.996	0.996
1.90	0.995	0.996	0.996	0.997	0.997	0.997	0.998	0.998
1.95	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999
2.00	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
2.05	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.10	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	61	62	63	64	65	66	67	68
1.00	0.163	0.164	0.164	0.164	0.164	0.165	0.165	0.165
1.05	0.238	0.239	0.240	0.241	0.242	0.243	0.244	0.245
1.10	0.326	0.328	0.330	0.332	0.334	0.335	0.337	0.339
1.15	0.421	0.424	0.427	0.430	0.433	0.435	0.438	0.441
1.20	0.518	0.522	0.526	0.529	0.533	0.537	0.540	0.544
1.25	0.612	0.616	0.620	0.625	0.629	0.633	0.637	0.641
1.30	0.697	0.701	0.706	0.710	0.715	0.719	0.724	0.728
1.35	0.770	0.775	0.779	0.784	0.788	0.792	0.797	0.801
1.40	0.831	0.835	0.840	0.844	0.848	0.851	0.855	0.859
1.45	0.879	0.883	0.887	0.890	0.894	0.897	0.900	0.903
1.50	0.916	0.919	0.922	0.925	0.928	0.931	0.933	0.936
1.55	0.943	0.946	0.948	0.950	0.953	0.955	0.957	0.959
1.60	0.963	0.964	0.966	0.968	0.970	0.971	0.973	0.974
1.65	0.976	0.977	0.979	0.980	0.981	0.982	0.983	0.984
1.70	0.985	0.986	0.987	0.988	0.988	0.989	0.990	0.991
1.75	0.991	0.991	0.992	0.992	0.993	0.994	0.994	0.994
1.80	0.994	0.995	0.995	0.996	0.996	0.996	0.997	0.997
1.85	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998
1.90	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999
1.95	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2.00	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	69	70	71	72	73	74	75	76
1.00	0.165	0.166	0.166	0.166	0.166	0.167	0.167	0.167
1.05	0.246	0.247	0.248	0.249	0.250	0.251	0.252	0.253
1.10	0.341	0.343	0.345	0.347	0.348	0.350	0.352	0.354
1.15	0.444	0.447	0.449	0.452	0.455	0.457	0.460	0.463
1.20	0.548	0.551	0.555	0.558	0.561	0.565	0.568	0.571
1.25	0.645	0.649	0.653	0.657	0.661	0.665	0.668	0.672
1.30	0.732	0.736	0.740	0.744	0.748	0.752	0.756	0.759
1.35	0.805	0.809	0.812	0.816	0.820	0.823	0.827	0.830
1.40	0.862	0.866	0.869	0.873	0.876	0.879	0.882	0.885
1.45	0.906	0.909	0.912	0.915	0.917	0.920	0.923	0.925
1.50	0.938	0.940	0.943	0.945	0.947	0.949	0.951	0.953
1.55	0.960	0.962	0.964	0.965	0.967	0.968	0.970	0.971
1.60	0.975	0.977	0.978	0.979	0.980	0.981	0.982	0.983
1.65	0.985	0.986	0.987	0.988	0.988	0.989	0.990	0.990
1.70	0.991	0.992	0.992	0.993	0.993	0.994	0.994	0.995
1.75	0.995	0.995	0.996	0.996	0.996	0.997	0.997	0.997
1.80	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998
1.85	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.90	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	77	78	79	80	81	82	83	84
1.00	0.167	0.167	0.168	0.168	0.168	0.168	0.168	0.169
1.05	0.254	0.254	0.255	0.256	0.257	0.258	0.259	0.260
1.10	0.355	0.357	0.359	0.361	0.362	0.364	0.366	0.367
1.15	0.465	0.468	0.471	0.473	0.476	0.478	0.481	0.483
1.20	0.575	0.578	0.581	0.584	0.588	0.591	0.594	0.597
1.25	0.676	0.679	0.683	0.687	0.690	0.693	0.697	0.700
1.30	0.763	0.767	0.770	0.774	0.777	0.781	0.784	0.787
1.35	0.834	0.837	0.840	0.844	0.847	0.850	0.853	0.856
1.40	0.888	0.891	0.894	0.896	0.899	0.901	0.904	0.906
1.45	0.927	0.930	0.932	0.934	0.936	0.938	0.940	0.942
1.50	0.954	0.956	0.958	0.959	0.961	0.962	0.964	0.965
1.55	0.972	0.974	0.975	0.976	0.977	0.978	0.979	0.980
1.60	0.984	0.985	0.985	0.986	0.987	0.988	0.988	0.989
1.65	0.991	0.991	0.992	0.992	0.993	0.993	0.994	0.994
1.70	0.995	0.995	0.996	0.996	0.996	0.996	0.997	0.997
1.75	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998
1.80	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.85	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	85	86	87	88	89	90	91	92
1.00	0.169	0.169	0.169	0.169	0.169	0.170	0.170	0.170
1.05	0.260	0.261	0.262	0.263	0.264	0.265	0.265	0.266
1.10	0.369	0.371	0.372	0.374	0.376	0.377	0.379	0.380
1.15	0.486	0.488	0.491	0.493	0.495	0.498	0.500	0.502
1.20	0.600	0.603	0.606	0.609	0.612	0.615	0.618	0.621
1.25	0.704	0.707	0.710	0.713	0.716	0.720	0.723	0.726
1.30	0.790	0.794	0.797	0.800	0.803	0.806	0.809	0.812
1.35	0.859	0.861	0.864	0.867	0.870	0.872	0.875	0.877
1.40	0.909	0.911	0.913	0.916	0.918	0.920	0.922	0.924
1.45	0.944	0.945	0.947	0.949	0.950	0.952	0.953	0.955
1.50	0.966	0.968	0.969	0.970	0.971	0.972	0.973	0.974
1.55	0.981	0.982	0.982	0.983	0.984	0.985	0.985	0.986
1.60	0.989	0.990	0.990	0.991	0.991	0.992	0.992	0.993
1.65	0.994	0.995	0.995	0.995	0.996	0.996	0.996	0.996
1.70	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998
1.75	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.80	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
1.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 80 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	93	94	95	96	97	98	99	100
1.00	0.170	0.170	0.170	0.170	0.171	0.171	0.171	0.171
1.05	0.267	0.268	0.269	0.269	0.270	0.271	0.272	0.272
1.10	0.382	0.384	0.385	0.387	0.388	0.390	0.391	0.393
1.15	0.505	0.507	0.509	0.512	0.514	0.516	0.519	0.521
1.20	0.624	0.626	0.629	0.632	0.635	0.638	0.640	0.643
1.25	0.729	0.732	0.735	0.738	0.741	0.743	0.746	0.749
1.30	0.815	0.818	0.820	0.823	0.826	0.828	0.831	0.834
1.35	0.880	0.882	0.885	0.887	0.889	0.891	0.893	0.896
1.40	0.926	0.928	0.929	0.931	0.933	0.935	0.936	0.938
1.45	0.956	0.958	0.959	0.960	0.961	0.963	0.964	0.965
1.50	0.975	0.976	0.977	0.978	0.979	0.980	0.980	0.981
1.55	0.987	0.987	0.988	0.988	0.989	0.989	0.990	0.990
1.60	0.993	0.993	0.994	0.994	0.994	0.995	0.995	0.995
1.65	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998
1.70	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
1.75	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
1.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE FOR
90 PERCENT CONFIDENCE

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	5	6	7	8	9	10	11	12
1.00	0.036	0.044	0.049	0.053	0.055	0.057	0.059	0.060
1.05	0.043	0.052	0.059	0.064	0.067	0.071	0.073	0.076
1.10	0.050	0.062	0.070	0.076	0.081	0.085	0.089	0.093
1.15	0.059	0.072	0.082	0.089	0.096	0.102	0.107	0.112
1.20	0.068	0.083	0.095	0.104	0.113	0.120	0.127	0.134
1.25	0.078	0.095	0.109	0.120	0.130	0.140	0.149	0.157
1.30	0.088	0.108	0.124	0.137	0.150	0.161	0.172	0.182
1.35	0.099	0.121	0.140	0.156	0.170	0.184	0.197	0.209
1.40	0.111	0.136	0.156	0.175	0.192	0.208	0.223	0.238
1.45	0.124	0.151	0.174	0.195	0.214	0.233	0.250	0.267
1.50	0.136	0.166	0.192	0.216	0.238	0.258	0.278	0.298
1.55	0.150	0.183	0.211	0.237	0.262	0.285	0.307	0.329
1.60	0.164	0.199	0.231	0.260	0.287	0.312	0.337	0.361
1.65	0.178	0.217	0.251	0.282	0.312	0.340	0.367	0.393
1.70	0.193	0.234	0.271	0.305	0.337	0.368	0.397	0.425
1.75	0.208	0.252	0.292	0.329	0.363	0.396	0.427	0.456
1.80	0.223	0.270	0.313	0.352	0.389	0.424	0.457	0.488
1.85	0.238	0.289	0.334	0.376	0.415	0.451	0.486	0.518
1.90	0.254	0.307	0.355	0.399	0.440	0.479	0.515	0.548
1.95	0.270	0.326	0.376	0.423	0.465	0.505	0.543	0.578
2.00	0.286	0.345	0.398	0.446	0.490	0.532	0.570	0.606
2.05	0.302	0.364	0.419	0.469	0.515	0.557	0.596	0.633
2.10	0.318	0.382	0.439	0.491	0.539	0.582	0.622	0.658
2.15	0.334	0.401	0.460	0.513	0.562	0.606	0.646	0.683
2.20	0.351	0.419	0.480	0.535	0.585	0.629	0.670	0.706
2.25	0.367	0.437	0.500	0.556	0.606	0.652	0.692	0.729
2.30	0.383	0.456	0.520	0.577	0.628	0.673	0.714	0.749
2.35	0.399	0.473	0.539	0.597	0.648	0.694	0.734	0.769
2.40	0.415	0.491	0.557	0.616	0.668	0.713	0.753	0.787
2.45	0.430	0.508	0.576	0.635	0.686	0.732	0.771	0.805
2.50	0.446	0.525	0.593	0.653	0.705	0.749	0.788	0.821
2.55	0.461	0.541	0.611	0.670	0.722	0.766	0.803	0.835
2.60	0.476	0.558	0.627	0.687	0.738	0.782	0.818	0.849
2.65	0.491	0.573	0.644	0.703	0.754	0.796	0.832	0.862
2.70	0.505	0.589	0.659	0.719	0.769	0.810	0.845	0.874
2.75	0.520	0.604	0.674	0.733	0.783	0.824	0.857	0.885
2.80	0.534	0.618	0.689	0.748	0.796	0.836	0.868	0.895
2.85	0.547	0.633	0.703	0.761	0.809	0.847	0.879	0.904
2.90	0.561	0.646	0.717	0.774	0.821	0.858	0.889	0.913
2.95	0.574	0.660	0.730	0.786	0.832	0.868	0.897	0.920
3.00	0.587	0.673	0.742	0.798	0.842	0.878	0.906	0.927

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	13	14	15	16	17	18	19	20
1.00	0.062	0.063	0.064	0.065	0.066	0.066	0.067	0.068
1.05	0.078	0.080	0.082	0.083	0.085	0.087	0.088	0.090
1.10	0.096	0.099	0.102	0.105	0.108	0.111	0.113	0.116
1.15	0.117	0.121	0.126	0.130	0.134	0.138	0.142	0.146
1.20	0.140	0.146	0.152	0.158	0.164	0.169	0.174	0.180
1.25	0.165	0.173	0.181	0.188	0.196	0.203	0.210	0.217
1.30	0.192	0.202	0.212	0.221	0.230	0.239	0.248	0.257
1.35	0.221	0.233	0.245	0.256	0.267	0.278	0.289	0.300
1.40	0.252	0.266	0.280	0.293	0.306	0.319	0.331	0.344
1.45	0.284	0.300	0.316	0.331	0.346	0.361	0.375	0.389
1.50	0.317	0.335	0.353	0.370	0.387	0.403	0.419	0.435
1.55	0.350	0.370	0.390	0.409	0.428	0.446	0.463	0.480
1.60	0.384	0.406	0.427	0.448	0.468	0.488	0.507	0.525
1.65	0.418	0.442	0.465	0.487	0.508	0.529	0.549	0.568
1.70	0.451	0.477	0.502	0.525	0.548	0.569	0.590	0.610
1.75	0.485	0.512	0.537	0.562	0.585	0.608	0.629	0.649
1.80	0.517	0.546	0.572	0.598	0.622	0.644	0.666	0.686
1.85	0.549	0.578	0.606	0.632	0.656	0.679	0.701	0.721
1.90	0.580	0.610	0.638	0.664	0.689	0.711	0.733	0.753
1.95	0.610	0.640	0.669	0.695	0.719	0.742	0.763	0.782
2.00	0.639	0.669	0.697	0.723	0.747	0.770	0.790	0.809
2.05	0.666	0.696	0.725	0.750	0.774	0.795	0.815	0.833
2.10	0.692	0.722	0.750	0.775	0.798	0.819	0.837	0.854
2.15	0.716	0.746	0.773	0.798	0.820	0.840	0.858	0.873
2.20	0.739	0.769	0.795	0.819	0.840	0.859	0.876	0.891
2.25	0.761	0.790	0.816	0.838	0.858	0.876	0.892	0.906
2.30	0.781	0.809	0.834	0.856	0.875	0.892	0.906	0.919
2.35	0.800	0.827	0.851	0.872	0.890	0.905	0.919	0.930
2.40	0.818	0.844	0.866	0.886	0.903	0.917	0.930	0.940
2.45	0.834	0.859	0.881	0.899	0.915	0.928	0.940	0.949
2.50	0.849	0.873	0.893	0.911	0.925	0.938	0.948	0.957
2.55	0.863	0.886	0.905	0.921	0.935	0.946	0.955	0.963
2.60	0.875	0.897	0.915	0.930	0.943	0.953	0.962	0.969
2.65	0.887	0.908	0.925	0.939	0.950	0.960	0.967	0.974
2.70	0.898	0.917	0.933	0.946	0.957	0.965	0.972	0.978
2.75	0.907	0.926	0.941	0.953	0.962	0.970	0.976	0.981
2.80	0.916	0.934	0.947	0.958	0.967	0.974	0.980	0.984
2.85	0.924	0.941	0.953	0.964	0.972	0.978	0.983	0.987
2.90	0.932	0.947	0.959	0.968	0.975	0.981	0.985	0.989
2.95	0.938	0.952	0.963	0.972	0.979	0.984	0.988	0.991
3.00	0.944	0.958	0.968	0.975	0.981	0.986	0.989	0.992

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	21	22	23	24	25	26	27	28
1.00	0.068	0.069	0.070	0.070	0.071	0.071	0.072	0.072
1.05	0.091	0.093	0.094	0.095	0.096	0.098	0.099	0.100
1.10	0.118	0.121	0.123	0.125	0.127	0.130	0.132	0.134
1.15	0.150	0.153	0.157	0.160	0.164	0.167	0.170	0.174
1.20	0.185	0.190	0.195	0.200	0.205	0.209	0.214	0.219
1.25	0.224	0.230	0.237	0.243	0.250	0.256	0.262	0.269
1.30	0.266	0.274	0.282	0.290	0.298	0.306	0.314	0.322
1.35	0.310	0.320	0.330	0.340	0.349	0.359	0.368	0.378
1.40	0.356	0.368	0.379	0.391	0.402	0.413	0.424	0.434
1.45	0.403	0.416	0.429	0.442	0.455	0.467	0.479	0.491
1.50	0.450	0.465	0.479	0.494	0.507	0.521	0.534	0.547
1.55	0.497	0.513	0.528	0.544	0.558	0.573	0.587	0.600
1.60	0.542	0.560	0.576	0.592	0.607	0.622	0.637	0.650
1.65	0.587	0.604	0.621	0.638	0.654	0.669	0.683	0.697
1.70	0.629	0.647	0.664	0.681	0.697	0.712	0.726	0.740
1.75	0.668	0.687	0.704	0.721	0.736	0.751	0.765	0.779
1.80	0.706	0.724	0.741	0.757	0.772	0.787	0.800	0.813
1.85	0.740	0.758	0.774	0.790	0.805	0.818	0.831	0.843
1.90	0.771	0.789	0.805	0.820	0.833	0.846	0.858	0.869
1.95	0.800	0.816	0.832	0.846	0.859	0.871	0.882	0.892
2.00	0.826	0.841	0.856	0.869	0.881	0.892	0.902	0.911
2.05	0.849	0.864	0.877	0.889	0.900	0.910	0.919	0.927
2.10	0.869	0.883	0.896	0.907	0.917	0.926	0.934	0.941
2.15	0.888	0.900	0.912	0.922	0.931	0.939	0.946	0.952
2.20	0.904	0.915	0.926	0.935	0.943	0.950	0.956	0.962
2.25	0.918	0.928	0.938	0.946	0.953	0.959	0.965	0.969
2.30	0.930	0.940	0.948	0.955	0.961	0.967	0.972	0.976
2.35	0.940	0.949	0.957	0.963	0.968	0.973	0.977	0.981
2.40	0.950	0.957	0.964	0.970	0.974	0.978	0.982	0.985
2.45	0.957	0.964	0.970	0.975	0.979	0.983	0.985	0.988
2.50	0.964	0.970	0.975	0.980	0.983	0.986	0.988	0.990
2.55	0.970	0.975	0.980	0.983	0.986	0.989	0.991	0.993
2.60	0.975	0.979	0.983	0.986	0.989	0.991	0.993	0.994
2.65	0.979	0.983	0.986	0.989	0.991	0.993	0.994	0.995
2.70	0.982	0.986	0.989	0.991	0.993	0.994	0.996	0.996
2.75	0.985	0.988	0.991	0.993	0.994	0.996	0.996	0.997
2.80	0.988	0.990	0.992	0.994	0.995	0.996	0.997	0.998
2.85	0.990	0.992	0.994	0.995	0.996	0.997	0.998	0.998
2.90	0.991	0.993	0.995	0.996	0.997	0.998	0.998	0.999
2.95	0.993	0.995	0.996	0.997	0.998	0.998	0.999	0.999
3.00	0.994	0.996	0.997	0.998	0.998	0.999	0.999	0.999

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	29	30	31	32	33	34	35	36
1.00	0.072	0.073	0.073	0.074	0.074	0.074	0.075	0.075
1.05	0.101	0.102	0.103	0.104	0.105	0.106	0.107	0.108
1.10	0.136	0.138	0.140	0.142	0.144	0.146	0.147	0.149
1.15	0.177	0.180	0.183	0.186	0.189	0.192	0.195	0.198
1.20	0.223	0.228	0.232	0.237	0.241	0.246	0.250	0.254
1.25	0.275	0.281	0.287	0.292	0.298	0.304	0.310	0.315
1.30	0.329	0.337	0.344	0.352	0.359	0.366	0.373	0.380
1.35	0.387	0.396	0.404	0.413	0.422	0.430	0.439	0.447
1.40	0.445	0.455	0.465	0.475	0.485	0.495	0.504	0.513
1.45	0.503	0.514	0.525	0.536	0.547	0.557	0.568	0.578
1.50	0.559	0.571	0.583	0.595	0.606	0.617	0.628	0.638
1.55	0.613	0.626	0.638	0.650	0.662	0.673	0.684	0.695
1.60	0.664	0.677	0.689	0.701	0.713	0.724	0.735	0.745
1.65	0.711	0.723	0.736	0.748	0.759	0.770	0.780	0.790
1.70	0.753	0.766	0.778	0.789	0.800	0.810	0.820	0.829
1.75	0.791	0.803	0.815	0.825	0.835	0.845	0.854	0.863
1.80	0.825	0.836	0.847	0.857	0.866	0.875	0.883	0.891
1.85	0.854	0.865	0.874	0.883	0.892	0.900	0.907	0.914
1.90	0.880	0.889	0.898	0.906	0.913	0.920	0.927	0.933
1.95	0.901	0.910	0.918	0.925	0.931	0.937	0.943	0.948
2.00	0.919	0.927	0.934	0.940	0.946	0.951	0.956	0.960
2.05	0.935	0.941	0.947	0.953	0.958	0.962	0.966	0.970
2.10	0.947	0.953	0.958	0.963	0.967	0.971	0.974	0.977
2.15	0.958	0.963	0.967	0.971	0.975	0.978	0.980	0.983
2.20	0.967	0.971	0.974	0.978	0.981	0.983	0.985	0.987
2.25	0.973	0.977	0.980	0.983	0.985	0.987	0.989	0.991
2.30	0.979	0.982	0.985	0.987	0.989	0.990	0.992	0.993
2.35	0.984	0.986	0.988	0.990	0.992	0.993	0.994	0.995
2.40	0.987	0.989	0.991	0.992	0.994	0.995	0.996	0.996
2.45	0.990	0.992	0.993	0.994	0.995	0.996	0.997	0.997
2.50	0.992	0.994	0.995	0.996	0.996	0.997	0.998	0.998
2.55	0.994	0.995	0.996	0.997	0.997	0.998	0.998	0.999
2.60	0.995	0.996	0.997	0.998	0.998	0.998	0.999	0.999
2.65	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999
2.70	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999
2.75	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000
2.80	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.85	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.90	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.95	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
3.00	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	37	38	39	40	41	42	43	44
1.00	0.075	0.075	0.076	0.076	0.076	0.077	0.077	0.077
1.05	0.109	0.110	0.111	0.112	0.113	0.113	0.114	0.115
1.10	0.151	0.153	0.155	0.156	0.158	0.160	0.162	0.163
1.15	0.201	0.204	0.207	0.210	0.213	0.215	0.218	0.221
1.20	0.258	0.262	0.267	0.271	0.275	0.279	0.283	0.287
1.25	0.321	0.326	0.332	0.337	0.343	0.348	0.353	0.358
1.30	0.387	0.394	0.401	0.407	0.414	0.421	0.427	0.433
1.35	0.455	0.463	0.471	0.479	0.486	0.494	0.501	0.509
1.40	0.522	0.531	0.540	0.549	0.557	0.566	0.574	0.582
1.45	0.587	0.597	0.606	0.616	0.625	0.633	0.642	0.651
1.50	0.649	0.659	0.668	0.678	0.687	0.696	0.705	0.713
1.55	0.705	0.715	0.724	0.734	0.743	0.752	0.760	0.768
1.60	0.755	0.765	0.774	0.783	0.792	0.800	0.808	0.816
1.65	0.800	0.809	0.818	0.826	0.834	0.842	0.849	0.856
1.70	0.838	0.847	0.855	0.862	0.870	0.877	0.883	0.889
1.75	0.871	0.878	0.886	0.892	0.899	0.905	0.911	0.916
1.80	0.898	0.905	0.911	0.917	0.923	0.928	0.933	0.937
1.85	0.920	0.926	0.932	0.937	0.941	0.946	0.950	0.954
1.90	0.938	0.943	0.948	0.952	0.956	0.960	0.963	0.966
1.95	0.953	0.957	0.961	0.964	0.967	0.970	0.973	0.976
2.00	0.964	0.967	0.971	0.974	0.976	0.978	0.981	0.983
2.05	0.973	0.976	0.978	0.981	0.983	0.984	0.986	0.988
2.10	0.980	0.982	0.984	0.986	0.987	0.989	0.990	0.991
2.15	0.985	0.987	0.988	0.990	0.991	0.992	0.993	0.994
2.20	0.989	0.990	0.992	0.993	0.994	0.994	0.995	0.996
2.25	0.992	0.993	0.994	0.995	0.996	0.996	0.997	0.997
2.30	0.994	0.995	0.996	0.996	0.997	0.997	0.998	0.998
2.35	0.996	0.996	0.997	0.997	0.998	0.998	0.998	0.999
2.40	0.997	0.997	0.998	0.998	0.998	0.999	0.999	0.999
2.45	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999
2.50	0.998	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.55	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
2.60	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.65	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	45	46	47	48	49	50	51	52
1.00	0.077	0.077	0.078	0.078	0.078	0.078	0.078	0.079
1.05	0.116	0.117	0.117	0.115	0.119	0.120	0.121	0.121
1.10	0.165	0.167	0.168	0.170	0.171	0.173	0.175	0.176
1.15	0.224	0.226	0.229	0.232	0.234	0.237	0.240	0.242
1.20	0.291	0.295	0.298	0.302	0.306	0.310	0.314	0.317
1.25	0.364	0.369	0.374	0.379	0.384	0.389	0.394	0.399
1.30	0.440	0.446	0.452	0.458	0.464	0.470	0.476	0.482
1.35	0.516	0.523	0.530	0.537	0.544	0.551	0.558	0.564
1.40	0.590	0.598	0.605	0.613	0.620	0.627	0.635	0.642
1.45	0.659	0.667	0.675	0.683	0.690	0.698	0.705	0.712
1.50	0.721	0.729	0.737	0.745	0.752	0.759	0.767	0.773
1.55	0.776	0.784	0.792	0.799	0.806	0.813	0.819	0.825
1.60	0.824	0.831	0.838	0.844	0.851	0.857	0.863	0.868
1.65	0.863	0.870	0.876	0.882	0.887	0.892	0.898	0.902
1.70	0.895	0.901	0.906	0.911	0.916	0.921	0.925	0.929
1.75	0.921	0.926	0.931	0.935	0.939	0.943	0.946	0.949
1.80	0.941	0.945	0.949	0.953	0.956	0.959	0.962	0.964
1.85	0.957	0.960	0.963	0.966	0.969	0.971	0.973	0.975
1.90	0.969	0.972	0.974	0.976	0.978	0.980	0.982	0.983
1.95	0.978	0.980	0.982	0.983	0.985	0.986	0.988	0.989
2.00	0.984	0.986	0.987	0.989	0.990	0.991	0.992	0.993
2.05	0.989	0.990	0.991	0.992	0.993	0.994	0.994	0.995
2.10	0.992	0.993	0.994	0.995	0.995	0.996	0.996	0.997
2.15	0.995	0.995	0.996	0.996	0.997	0.997	0.998	0.998
2.20	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999
2.25	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
2.30	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2.35	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
2.40	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	53	54	55	56	57	58	59	60
1.00	0.079	0.079	0.079	0.079	0.079	0.080	0.080	0.080
1.05	0.122	0.123	0.123	0.124	0.125	0.126	0.126	0.127
1.10	0.178	0.179	0.181	0.182	0.184	0.185	0.187	0.188
1.15	0.245	0.247	0.250	0.252	0.255	0.257	0.260	0.262
1.20	0.321	0.325	0.328	0.332	0.336	0.339	0.343	0.346
1.25	0.404	0.408	0.413	0.418	0.422	0.427	0.432	0.436
1.30	0.488	0.494	0.499	0.505	0.511	0.516	0.522	0.527
1.35	0.571	0.577	0.584	0.590	0.596	0.602	0.608	0.614
1.40	0.649	0.655	0.662	0.669	0.675	0.681	0.687	0.694
1.45	0.719	0.726	0.732	0.739	0.745	0.751	0.757	0.763
1.50	0.780	0.786	0.793	0.799	0.805	0.811	0.816	0.822
1.55	0.832	0.838	0.843	0.849	0.854	0.859	0.864	0.869
1.60	0.874	0.879	0.884	0.889	0.893	0.898	0.902	0.906
1.65	0.907	0.912	0.916	0.920	0.924	0.927	0.931	0.934
1.70	0.933	0.937	0.940	0.943	0.947	0.950	0.952	0.955
1.75	0.953	0.955	0.958	0.961	0.963	0.966	0.968	0.970
1.80	0.967	0.969	0.971	0.973	0.975	0.977	0.979	0.980
1.85	0.977	0.979	0.981	0.982	0.984	0.985	0.986	0.987
1.90	0.985	0.986	0.987	0.988	0.989	0.990	0.991	0.992
1.95	0.990	0.991	0.992	0.992	0.993	0.994	0.994	0.995
2.00	0.993	0.994	0.995	0.995	0.996	0.996	0.996	0.997
2.05	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.998
2.10	0.997	0.998	0.998	0.998	0.998	0.999	0.999	0.999
2.15	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
2.20	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.25	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	61	62	63	64	65	66	67	68
1.00	0.080	0.080	0.080	0.081	0.081	0.081	0.081	0.081
1.05	0.128	0.128	0.129	0.130	0.130	0.131	0.131	0.132
1.10	0.190	0.191	0.192	0.194	0.195	0.197	0.198	0.200
1.15	0.265	0.267	0.270	0.272	0.274	0.277	0.279	0.281
1.20	0.350	0.353	0.357	0.360	0.364	0.367	0.371	0.374
1.25	0.441	0.445	0.450	0.454	0.459	0.463	0.467	0.472
1.30	0.532	0.538	0.543	0.548	0.553	0.558	0.563	0.568
1.35	0.620	0.626	0.631	0.637	0.642	0.648	0.653	0.659
1.40	0.700	0.705	0.711	0.717	0.722	0.728	0.733	0.739
1.45	0.769	0.775	0.780	0.786	0.791	0.796	0.801	0.806
1.50	0.827	0.832	0.837	0.842	0.847	0.852	0.856	0.860
1.55	0.874	0.878	0.883	0.887	0.891	0.895	0.899	0.902
1.60	0.910	0.914	0.918	0.921	0.924	0.928	0.931	0.934
1.65	0.938	0.941	0.944	0.946	0.949	0.951	0.954	0.956
1.70	0.958	0.960	0.962	0.964	0.966	0.968	0.970	0.972
1.75	0.972	0.974	0.975	0.977	0.978	0.980	0.981	0.982
1.80	0.982	0.983	0.984	0.985	0.986	0.987	0.988	0.989
1.85	0.988	0.989	0.990	0.991	0.991	0.992	0.993	0.993
1.90	0.993	0.993	0.994	0.994	0.995	0.995	0.996	0.996
1.95	0.995	0.996	0.996	0.997	0.997	0.997	0.997	0.998
2.00	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999
2.05	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
2.10	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
2.15	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	69	70	71	72	73	74	75	76
1.00	0.081	0.081	0.081	0.082	0.082	0.082	0.082	0.082
1.05	0.133	0.133	0.134	0.135	0.135	0.136	0.136	0.137
1.10	0.201	0.202	0.204	0.205	0.206	0.208	0.209	0.210
1.15	0.284	0.286	0.289	0.291	0.293	0.295	0.298	0.300
1.20	0.377	0.381	0.384	0.387	0.391	0.394	0.397	0.400
1.25	0.476	0.480	0.484	0.488	0.493	0.497	0.501	0.505
1.30	0.573	0.578	0.583	0.588	0.592	0.597	0.602	0.606
1.35	0.664	0.669	0.674	0.679	0.684	0.689	0.694	0.698
1.40	0.744	0.749	0.754	0.759	0.763	0.768	0.773	0.777
1.45	0.811	0.816	0.820	0.825	0.829	0.833	0.837	0.842
1.50	0.865	0.869	0.873	0.877	0.880	0.884	0.888	0.891
1.55	0.906	0.909	0.913	0.916	0.919	0.922	0.925	0.928
1.60	0.937	0.939	0.942	0.944	0.947	0.949	0.951	0.953
1.65	0.958	0.960	0.962	0.964	0.966	0.968	0.969	0.971
1.70	0.973	0.975	0.976	0.978	0.979	0.980	0.981	0.982
1.75	0.983	0.984	0.985	0.986	0.987	0.988	0.989	0.990
1.80	0.990	0.991	0.991	0.992	0.992	0.993	0.993	0.994
1.85	0.994	0.994	0.995	0.995	0.996	0.996	0.996	0.997
1.90	0.996	0.997	0.997	0.997	0.998	0.998	0.998	0.998
1.95	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
2.00	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2.05	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	77	78	79	80	81	82	83	84
1.00	0.082	0.082	0.082	0.082	0.082	0.083	0.083	0.083
1.05	0.138	0.138	0.139	0.139	0.140	0.141	0.141	0.142
1.10	0.212	0.213	0.214	0.216	0.217	0.218	0.220	0.221
1.15	0.302	0.304	0.307	0.309	0.311	0.313	0.316	0.318
1.20	0.404	0.407	0.410	0.413	0.416	0.419	0.422	0.426
1.25	0.509	0.513	0.517	0.521	0.525	0.528	0.532	0.536
1.30	0.611	0.615	0.620	0.624	0.628	0.633	0.637	0.641
1.35	0.703	0.708	0.712	0.717	0.721	0.725	0.730	0.734
1.40	0.782	0.786	0.790	0.795	0.799	0.803	0.807	0.811
1.45	0.845	0.849	0.853	0.857	0.860	0.864	0.867	0.871
1.50	0.894	0.898	0.901	0.904	0.907	0.910	0.913	0.915
1.55	0.930	0.933	0.935	0.938	0.940	0.942	0.944	0.946
1.60	0.955	0.957	0.959	0.961	0.963	0.964	0.966	0.967
1.65	0.972	0.974	0.975	0.976	0.977	0.979	0.980	0.981
1.70	0.983	0.984	0.985	0.986	0.987	0.988	0.988	0.989
1.75	0.990	0.991	0.991	0.992	0.993	0.993	0.993	0.994
1.80	0.994	0.995	0.995	0.996	0.996	0.996	0.996	0.997
1.85	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998
1.90	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
1.95	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000
2.00	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	85	86	87	88	89	90	91	92
1.00	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
1.05	0.142	0.143	0.143	0.144	0.144	0.145	0.146	0.146
1.10	0.222	0.224	0.225	0.226	0.227	0.229	0.230	0.231
1.15	0.320	0.322	0.324	0.326	0.329	0.331	0.333	0.335
1.20	0.429	0.432	0.435	0.438	0.441	0.444	0.447	0.450
1.25	0.540	0.544	0.547	0.551	0.555	0.558	0.562	0.565
1.30	0.645	0.649	0.653	0.657	0.661	0.665	0.669	0.673
1.35	0.738	0.742	0.746	0.750	0.754	0.758	0.762	0.765
1.40	0.815	0.818	0.822	0.826	0.829	0.833	0.836	0.840
1.45	0.874	0.877	0.880	0.884	0.887	0.889	0.892	0.895
1.50	0.918	0.920	0.923	0.925	0.928	0.930	0.932	0.934
1.55	0.948	0.950	0.952	0.954	0.956	0.957	0.959	0.961
1.60	0.969	0.970	0.971	0.973	0.974	0.975	0.976	0.977
1.65	0.982	0.983	0.984	0.984	0.985	0.986	0.987	0.987
1.70	0.990	0.990	0.991	0.991	0.992	0.992	0.993	0.993
1.75	0.994	0.995	0.995	0.995	0.996	0.996	0.996	0.997
1.80	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998
1.85	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
1.90	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PROBABILITY OF DEMONSTRATING TECHNICAL REQUIREMENT
WITH 90 PERCENT CONFIDENCE

EXPECTED NUMBER OF FAILURES

M(T)/TR	93	94	95	96	97	98	99	100
1.00	0.084	0.084	0.084	0.084	0.084	0.084	0.084	0.084
1.05	0.147	0.147	0.148	0.148	0.149	0.149	0.150	0.150
1.10	0.232	0.234	0.235	0.236	0.237	0.239	0.240	0.241
1.15	0.337	0.339	0.341	0.343	0.345	0.348	0.350	0.352
1.20	0.453	0.456	0.458	0.461	0.464	0.467	0.470	0.473
1.25	0.569	0.573	0.576	0.580	0.583	0.586	0.590	0.593
1.30	0.677	0.681	0.684	0.688	0.692	0.695	0.699	0.702
1.35	0.769	0.773	0.776	0.780	0.783	0.787	0.790	0.794
1.40	0.843	0.846	0.849	0.852	0.855	0.858	0.861	0.864
1.45	0.898	0.900	0.903	0.905	0.908	0.910	0.913	0.915
1.50	0.936	0.938	0.940	0.942	0.944	0.946	0.948	0.949
1.55	0.962	0.964	0.965	0.966	0.968	0.969	0.970	0.971
1.60	0.978	0.979	0.980	0.981	0.982	0.983	0.983	0.984
1.65	0.988	0.989	0.989	0.990	0.990	0.991	0.991	0.992
1.70	0.994	0.994	0.994	0.995	0.995	0.995	0.996	0.996
1.75	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998
1.80	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
1.85	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000
1.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.70	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.85	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.95	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

APPENDIX C
DERIVATIONS

Proposition 1.

$$f_{obs} \le c \quad \Leftrightarrow \quad TR \le \ell\gamma\,(f_{obs})$$

Proof.

To prove this relation, we use the equation below which follows directly from the definition of a 100 γ percent lower confidence bound when f_{obs} failures occur in a demonstration test of length T_{Dem}:

$$\sum_{i=0}^{f_{obs}} e^{-T_{Dem}/\ell} \frac{(T_{Dem}/\ell)^i}{i!} = 1-\gamma$$

where

$$\ell \quad \underline{\Delta} \quad \ell_\gamma\,(f_{obs}).$$

Let g be the function of x > 0 defined by the left-hand side of the equation above with ℓ replaced by x. Note g is a strictly increasing function of x > 0 since g(x) is the probability of obtaining f_{obs} or fewer failures when the constant configuration under test has MTBF x.

I. First we shall show $f_{obs} \le c \Rightarrow TR \le \ell$.

Thus, let $f_{obs} \le c$. Suppose ℓ < TR. Then

$$g(\ell) < g\,(TR) = \sum_{i=0}^{f_{obs}} e^{-T_{Dem}/TR} \frac{(T_{Dem}/TR)^i}{i!}$$

$$\le \sum_{i=0}^{c} e^{-T_{Dem}/TR} \frac{(T_{Dem}/TR)^i}{i!}$$

$$\le 1-\gamma$$

which is a contradiction since $g(\ell) = 1-\gamma$. Thus, $TR \le \ell$.

II. Next we shall show $TR \le \ell \Rightarrow f_{obs} \le c$. Thus, let $TR \le \ell$. Suppose $f_{obs} > c$. Then

$$\sum_{i=0}^{f_{obs}} e^{-T_{Dem}/TR} \frac{(T_{Dem}/TR)^i}{i!} \quad = \quad g(TR) \quad \leq \quad g(\ell) \quad = \quad 1-\gamma.$$

Since $f_{obs} > c$, this contradicts the definition of c (see Equation (5) in Section 2.1.2). Thus, $f_{obs} \leq c$.

<u>Proposition 2</u>.

For each α<1, T>0, and M(T)>0, the corresponding distribution function of $L_\gamma(N,S)$ satisfies the inequality

$$\mathrm{Pr}\,ob\left(L_\gamma(N,S) \leq M(T)\right) \quad \geq \quad \gamma$$

<u>Proof.</u>

Let f_W denote the density function of W (defined by Equation (20) in Section 2.1.3) corresponding to α<1, T>0, M(T)>0. By inequality (21) in Section 2.1.3,

$$\mathrm{Pr}\,ob\left(L_\gamma(N,S) \quad \leq \quad M(T)\right)$$

$$= \int_0^\infty \left\{\mathrm{Prob}\left(\mathrm{L}_\gamma(N,S;w) \leq M(T)\right)\right\} \mathrm{f_w}\,(\mathrm{w})\mathrm{dw}$$

$$\geq \quad \gamma \int_0^\infty \mathrm{f_w}\,(\mathrm{w})\,\mathrm{dw}$$

$$\geq \quad \gamma$$

<u>Proposition 3</u>.

For each α<1, T>0, and M(T)>0,

$$\mathrm{Pr}\,ob\left(L_\gamma(N,S) = x\right) \quad = \quad 0$$

for all real x.

<u>Proof.</u>

Let α<1, T>0, and M(T)>0. Clearly, $L_\gamma(N,S) \geq 0$. Thus, we need to consider $x \geq 0$.

Let $L_\gamma(n,S)$ denote $L_\gamma(N,S)$ conditioned on $N = n$. As shown in Appendix A of Reference 7,

$$\frac{\hat{M}_n(T)}{M(T)} \sim \left(\frac{\lambda T^\beta}{2n^2}\right)\chi^2_{2n}$$

where χ^2_ν is the chi-square random variable with ν degrees of freedom.

Thus,

$$\hat{M}_{\mathrm{n}}(\mathrm{T}) \sim \left(\frac{1}{\lambda\beta \mathrm{T}^{\beta\text{-}1}}\right)\left(\frac{\lambda \mathrm{T}^\beta}{2\mathrm{n}^2}\right)\chi^2_{2\mathrm{n}}$$

$$=\left(\frac{\mathrm{T}}{2\beta\mathrm{n}^2}\right)\chi^2_{2\mathrm{n}}$$

Then, by (12) in Section 2.1.3,

$$L_\gamma(\mathrm{n,S}) \sim \left(\frac{2\mathrm{n}}{z_\gamma(n)}\right)^2\left(\frac{T}{2\beta n^2}\right)\chi^2_{2\mathrm{n}}$$

i.e.,

$$L_\gamma(n,S) \sim \left(\frac{2T}{\beta}\right)\left(\frac{\chi^2_{2n}}{z^2_\gamma(n)}\right) \tag{34}$$

Thus,

$$\mathrm{Prob}\ (L_\gamma(\mathrm{n,S}) = \mathrm{x}) = \mathrm{Prob}\left(\chi^2_{2\mathrm{n}} = \frac{\beta z^2_\gamma\ (\mathrm{n})\,\mathrm{x}}{2\mathrm{T}}\right) = 0$$

It then follows that,

$$\mathrm{Prob}\ (L_\gamma\ (\mathrm{N,S}) = \mathrm{x}) =$$

$$[\mathrm{Prob}(N=0)]^{-1}\sum_{n=1}^{\infty}[\mathrm{Prob}(L_\gamma(n,S)=x)]\,\mathrm{Prob}(N=n)$$

= 0, since Prob (N=0) > 0.

<u>Proposition 4</u>.

Type II = $\Pr ob\left(TR \le L_\gamma(N,S)\right) \le 1-\gamma$ for each $\alpha < 1$ and $T > 0$ where $M(T) = TR$.

Proof.

Let $\alpha < 1$ and $T > 0$ with $M(T) = TR$.

Then

$$\Pr ob\left(TR \le L_\gamma(N,S)\right) =$$

$$\Pr ob\left(L_\gamma(N,S) = TR\right) \quad + \quad \Pr ob\left(TR < L_\gamma(N,S)\right)$$

$$= \Pr ob\left(TR < L_\gamma(N,S)\right), \text{ by Proposition 3,}$$

$$= 1 - \Pr ob\left(L_\gamma(N,S) \le TR\right) \quad \le \quad 1-\gamma, \text{ by Proposition 2.}$$

Proposition 5.

For a growth curve with parameters (α, T, M(T)), the expected number of failures (E(N)) can be determined by

$$\mathrm{E(N)} \quad = \quad \frac{\mathrm{T}}{(1-\alpha)\,\mathrm{M(T)}}$$

Proof.

The observed number of failures by test duration t, denoted by N(t), is a non-homogeneous Poisson process with $N(T) = N$ and intensity function

$$\mathrm{p(t)} \quad = \quad \frac{1}{\mathrm{M(T)}} \quad = \quad \lambda\beta t^{\beta-1}$$

This implies that N is Poisson distributed with expected value

$$\mathrm{E(N)} \quad = \quad \int_0^{\mathrm{T}} \rho(\mathrm{t})\,\mathrm{dt} \quad = \quad \lambda \mathrm{T}^{\beta}$$

By Equation (18) in Section 2.1.3,

$$\mathrm{E(N)} \quad = \quad \frac{\mathrm{T}^{\beta}}{(\mathrm{M(T)})\beta\mathrm{T}^{\beta-1}}$$

This yields

$$E(N) = \frac{T}{\beta M(T)} = \frac{\mathrm{T}}{(1-\alpha)(\mathrm{M(T)}}.$$

<u>Proposition 6</u>.

For a growth curve with parameters (α, T, M(T)),

Prob (A; α, T, M(T)) =

$$(1-e^{-\mu})^{-1} \sum_{n=1}^{\infty} \left[\mathrm{Prob} \left(\frac{\chi^2_{2n}}{z^2_{\gamma}\ (n)} \geq \frac{1}{2\mu d} \right) \right] e^{-\mu} \left(\frac{\mu^n}{n!} \right)$$

where $\mu \,\underline{\Delta}\, E(N)$ and $d \,\underline{\Delta}\, M(T)/TR$.

<u>Proof.</u>

From (23) in Section 2.1.3 and (34),

$$\Pr ob(A;\alpha,T,M(T)) = \Pr ob(L_{\gamma}(N,S) \geq TR)$$

$$= [1 - \mathrm{Prob}\ (\mathrm{N}=0)]^{-1} \sum_{n=1}^{\infty} [\mathrm{Prob}\ (\mathrm{L}\gamma\ (\mathrm{n,S}) \geq \mathrm{TR})]\ \mathrm{Prob}\ (\mathrm{N}=\mathrm{n})$$

$$= [1 - \Pr ob(N=0)]^{-1} \sum_{n=1}^{\infty} \left[\mathrm{Prob} \left(\left(\frac{2\mathrm{T}}{\beta} \right) \left(\frac{\chi^2_{2n}}{z^2_{\gamma}(n)} \right) \geq \mathrm{TR} \right) \right] \mathrm{Prob}\ (\mathrm{N}=\mathrm{n})$$

$$= [1 - \mathrm{Prob}\ (\mathrm{N}=0)]^{-1} \sum_{n=1}^{\infty} \left[\mathrm{Prob} \left(\frac{\chi^2_{2n}}{z^2_{\gamma}\ (\mathrm{x})} \geq \frac{\beta(\mathrm{TR})}{2\mathrm{T}} \right) \right] \mathrm{Prob}(\mathrm{N}=\mathrm{n})$$

Letting μ $\underline{\Delta}$ E(N) and d $\underline{\Delta}$ M(T)/TR,

$$\Pr ob(A;\alpha,T,M(T)) =$$

$$(1-e^{-\mu})^{-1} \sum_{n=1}^{\infty} \left[\mathrm{Prob} \left(\frac{\chi^2_{2n}}{z^2_{\gamma}(n)} \geq (1/2) \left(\frac{\beta \mathrm{M(T)}}{\mathrm{T}} \right) \left(\frac{TR}{M(T)} \right) \right) \right] e^{-\mu} \left(\frac{\mu^n}{n!} \right)$$

$$= \quad (1 - e^{-\mu})^{-1} \sum_{n=1}^{\infty} \left[\text{Prob}\left(\frac{\chi^2_{2n}}{z^2_{\gamma}\,(n)} \geq \frac{1}{2\mu d} \right) \right] e^{-\mu} \left(\frac{\mu^n}{n!} \right).$$

3 RELIABILITY GROWTH TRACKING

3.1 Introduction. This section contains material from MIL-HDBK-189 [1] on the AMSAA Continuous Tracking Model. In addition, it presents the AMSAA Discrete Tracking Model developed in [2] and an AMSAA subsystem level tracking model (SSTRACK), Reference [3].

3.1.1 Definition and Objectives of Reliability Growth Tracking. Reliability growth tracking is a process that allows management the opportunity to gauge the progress of the reliability effort for a system by obtaining a demonstrated numerical measure of the system reliability during a development program based on test data. Some objectives of reliability growth tracking include:

- determining if system reliability is increasing with time (i.e., growth is occurring) and to what degree (i.e., growth rate), and

- estimating the demonstrated reliability - a reliability estimate based on test data for the system configuration under test at the end of the test phase. This latter estimate is based on the actual performance of the system tested and not on some future configuration.

Reliability growth tracking allows for the situation where the configuration of the system may be changing as a result of the incorporation of corrective actions to problem failure modes. In the presence of reliability growth, the data from earlier configurations may not be representative of the current configuration of the system. On the other hand, the most recent test data, which would best represent the current system configuration, may be limited so that an estimate based upon the recent data would not, in itself, be sufficient for a valid determination of reliability. Because of this situation, reliability growth tracking may offer a viable method for combining test data from several configurations to obtain a demonstrated reliability estimate for the current system configuration, provided the reliability growth tracking model adequately represents the combined test data.

3.1.2 Managerial Role. The role of management in the reliability growth tracking process is twofold:

- to systematically plan and assess reliability achievement as a function of time and other program resources (such as personnel, money, available prototypes, etc.,) and,

- to control the ongoing rate of reliability achievement by the addition to or reallocation of these program resources based on comparisons between the planned and demonstrated reliability values.

To achieve reliability goals, it is important that the program manager be aware of reliability problems during the conduct of the development program so that effective system design changes can be funded and implemented. It is essential, therefore, that periodic assessments (tracking) of reliability be made during the test program (usually at the end of a test phase) and compared to the planned reliability goals. A comparison between the assessed and planned values will suggest whether the development program is progressing as planned, better than planned, or not as well as planned. Thus, tracking the improvement in system reliability through quantitative assessments of progress is an important management function.

3.1.3 Types of Reliability Growth Tracking Models. Reliability growth tracking models are distinguished according to the level at which testing is conducted and failure data are collected. They fall into two categories: system level and subsystem level. For system level reliability growth tracking models, testing is conducted in a full-up integrated manner, failure data are collected on an overall system basis, and an assessment is made regarding the system reliability. For subsystem level reliability growth tracking models, the subsystems are tested and the failure data are collected on an individual subsystem basis -- the subsystem data are then "rolled up" to arrive at an estimate for the demonstrated system reliability.

System level reliability growth tracking models are further classified according to the usage of the system. They fall into two groups -- continuous and discrete models -- and are defined by the type of outcome that the usage provides. Continuous models are those that apply to systems for which usage is measured on a continuous scale, such as time in hours or distance in miles. For continuous models, outcomes are usually measured in terms of an interval or range; for example, mean time/miles between failures. Discrete models are those that apply to systems for which usage is measured on an enumerative or classificatory basis, such as pass/fail or go/no-go. For discrete models, outcomes are recorded in terms of distinct, countable events that give rise to probability estimates.

3.1.4 Model Substitution.

List of Notation

Discrete Parameters:

N	number of trials = sample size
S	success
F	failure
NS	number of successes
NF	number of failures
U	unreliability
R	reliability

Continuous Parameters:

MTTF	mean time/trials to failure
MTBF	mean time/trials between failures

In general, continuous models are designed for continuous data, and discrete models are designed for discrete data. In the event a designated model is unavailable for use, it may be possible to use a continuous model for discrete data or a discrete model for continuous data. The latter case is generally not a practical option, though. (The AMSAA Subsystem Tracking Model, for example, is a continuous model that may be used with discrete data, subject to the conditions mentioned at the end of this paragraph.) In cases involving model substitution, the "substitute" model is used as an approximation for the intended model, and the original data appropriate for the intended model must be converted to a format appropriate for the substitute model. Note that in applying a continuous model to discrete data, the results of the approximation improve as the number of trials increases and the probability of failure decreases.

By way of an example, we show a method for converting discrete data to a continuous format and vice versa. Suppose that from a sample size of N = 5 trials the following outcomes are observed, where S denotes a success and F denotes a failure:

$$S \quad S \quad S \quad S \quad F$$

The number of successes, NS, is four; the number of failures, NF, is one; and $N = NS + NF$.

To begin, note that in discrete terms:

$$U \quad = \quad probability(failure) \quad = \quad \frac{NF}{N} \tag{1}$$

The reciprocal of U, namely N/NF, may be viewed as a measure of the number of trials to the number of failures, MTTF, thus allowing a continuous measure to be related to a discrete measure:

$$MTTF \quad = \quad \frac{1}{U} \tag{2}$$

In the example, MTTF = 5 and MTBF = 4, so that:

$$MTBF \quad = \quad MTTF - 1 \tag{3}$$

Substituting (2) into (3) and noting that $R = 1 - U$ results in:

$$MTBF \quad = \quad \frac{1}{U} - 1 \quad = \quad \frac{R}{1-R} \tag{4}$$

Equation (4) is used to convert a discrete measure to a continuous measure. To convert a continuous measure to a discrete measure, rearrange (4) and solve for R:

$$MTBF + 1 = \frac{1}{1 - R} \quad (5)$$

$$R = 1 - \frac{1}{MTBF + 1} \quad (6)$$

3.2 System Level Reliability Growth Tracking Models.

3.2.1 Continuous Tracking Models.

3.2.1.1 Background and Basis for the AMSAA Continuous Tracking Model.

List of Notation

t_i	cumulative test time when design modification i is made
K	final entry in a sequence of test times; point where the last design modification is made
λ_i	constant failure rate during i-th time interval
F_i	number of failures during i-th time interval
θ_i	mean value function for F_i
f	a particular realization of F_i
e	exponential function
t	cumulative test time
F(t)	total number of system failures by time t
$\theta(t)$	mean value function for F(t)
$\rho(y)$	failure rate for configuration i where $y \in [t_{i-1}, t_i)$
$\rho(t)$	instantaneous system failure rate at time t; also referred to as the failure intensity function
λ	scale parameter of parametric function $\rho(t)$; $(\lambda > 0)$
β	shape parameter of parametric function $\rho(t)$; $(\beta > 0)$
m(t)	instantaneous mean time between failures at time t
T	total test time
F	total observed number of failures by time T
X_i	cumulative time to i-th failure
^	denotes an estimate when placed over a parameter
L	lower confidence coefficient
U	upper confidence coefficient
γ	desired confidence level
-	denotes an unbiased estimate when placed over a parameter
α	significance level

The AMSAA Continuous Reliability Growth Tracking Model may be used to track the reliability improvement of a system during a development test phase for which usage is measured on a continuous scale. The model may also be used for tracking the reliability of one-shot (discrete) systems if there are a large number of trials and the system demonstrates high reliability during test.

The model is designed for tracking system reliability within a test phase and not across test phases. Accordingly, the basis of the model is described in the following way. Let the start of a test phase be initialized at time zero, and let $0 = t_0 < t_1 < t_2 < \ldots < t_K$ denote the cumulative test times on the system when design modifications are made. Assume the system failure rate is constant between successive $t_i's$, and let λ_i denote the constant failure rate during the i-th time interval $\left[t_{i-1}, t_i\right)$. The time intervals do not have to be equal in length. Based on the constant failure rate assumption, the number of failures F_i during the i-th time interval is Poisson distributed with mean $\theta_i = \lambda_i \left(t_i - t_{i-1}\right)$.

That is,

$$\Pr ob(F_i = f) \quad = \quad \frac{(\theta_i)^f \, e^{-\theta_i}}{f!} \qquad \left(f = 0,1,2,\ldots\right) \tag{7}$$

During developmental testing programs, if more than one system prototype is tested and if the prototypes have the same basic configuration between modifications, then under the constant failure rate assumption, the following are true:

- the time t_i may be considered as the cumulative test time to the i-th modification, and
- F_i may be considered as the cumulative total number of failures experienced by all system prototypes during the i-th time interval $\left[t_{i-1}, t_i\right)$.

The previous discussion is summarized graphically:

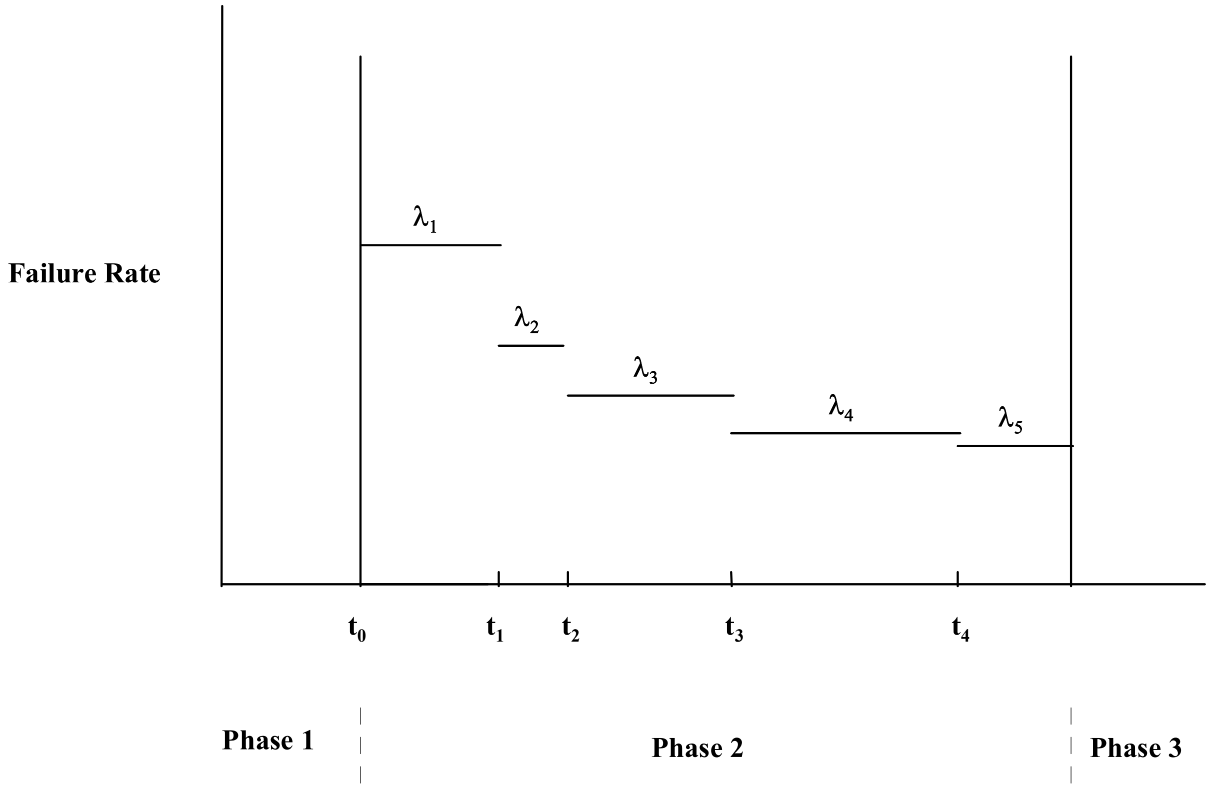

Figure 3.1 Failure Rates Between Modifications

Let t denote the cumulative test time, and let F(t) be the total number of system failures by time t. If t is in the first time interval:

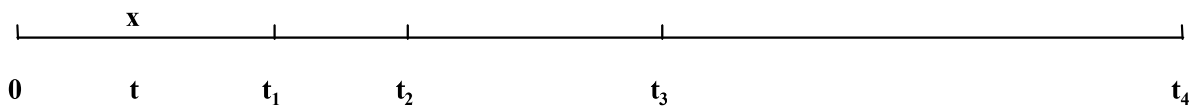

Figure 3.2 Time Line for Phase 2 (t in first time interval)

then F(t) has the Poisson distribution with mean $\lambda_1 t$. Now if t is in the second time interval:

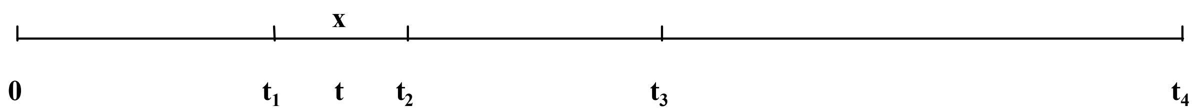

Figure 3.3 Time Line for Phase 2 (t in second time interval)

then F(t) is the number of system failures F_1 in the first time interval plus the number of system failures in the second time interval between t_1 and t. The failure rate for the first time interval is

λ_1, and the failure rate for the second time interval is λ_2. Therefore, the mean of F(t) is the sum of the mean of $F_1 = \lambda_1 t_1$ plus the mean number of failures from t_1 to t, which is $\lambda_2(t - t_1)$. That is, F(t) has mean $\theta(t) = \lambda_1 t_1 + \lambda_2(t - t_1)$.

When the failure rate is constant (homogeneous) over a test interval, then F(t) is said to follow a homogeneous Poisson process with mean number of failures of the form λt. When the failure rates change with time, e.g., from interval 1 to interval 2, then under certain conditions, F(t) is said to follow a nonhomogeneous Poisson process (NHPP). In the presence of reliability growth, F(t) would follow a NHPP with mean value function:

$$\theta(t) \quad = \quad \int_o^t \rho(y)dy \tag{8}$$

where $\rho(y) = \lambda_i$, $y \in [t_{i-1}, t_i)$. From (7), for any $t > 0$,

$$\Pr ob[F(t) = f] \quad = \quad \frac{[\theta(t)]^f e^{-\theta(t)}}{f!} \qquad (f = 0,1,2,...) \tag{9}$$

The integer-valued process $\{F(t), t > 0\}$ may be regarded as a NHPP with intensity function $\rho(t)$. The physical interpretation of $\rho(t)$ is that for infinitesimally small Δt, $\rho(t)\,\Delta t$ is approximately the probability of a system failure in the time interval $(t, t + \Delta t)$; that is, it is approximately the instantaneous system failure rate. If $\rho(t) = \lambda$, a constant failure rate for all t, then a system is experiencing no growth over time, corresponding to the exponential case. If $\rho(t)$ is decreasing with time, $(\lambda_1 > \lambda_2 > \lambda_3 \ldots)$, then a system is experiencing reliability growth. Finally, $\rho(t)$ increasing over time indicates deterioration in system reliability.

Based on the learning curve approach, which is outlined in detail in the section on the AMSAA Discrete Reliability Growth Tracking Model, the AMSAA Continuous Reliability Growth Tracking Model assumes that $\rho(t)$ may be approximated by a continuous, parametric function. Using a result established for the Discrete Model:

$$E[F(t)] \quad = \quad \lambda t^{\beta} \tag{10}$$

and the instantaneous system failure rate $\rho(t)$ is the change per unit time of E[F(t)]:

$$\rho(t) \quad = \quad \frac{d}{dt} E[F(t)] \quad = \quad \lambda\beta t^{\beta-1} \qquad (\lambda, \beta, t > 0) \tag{11}$$

With a failure rate $\rho(t)$ that may change with test time, the NHPP provides a basis for describing the reliability growth process within a test phase.

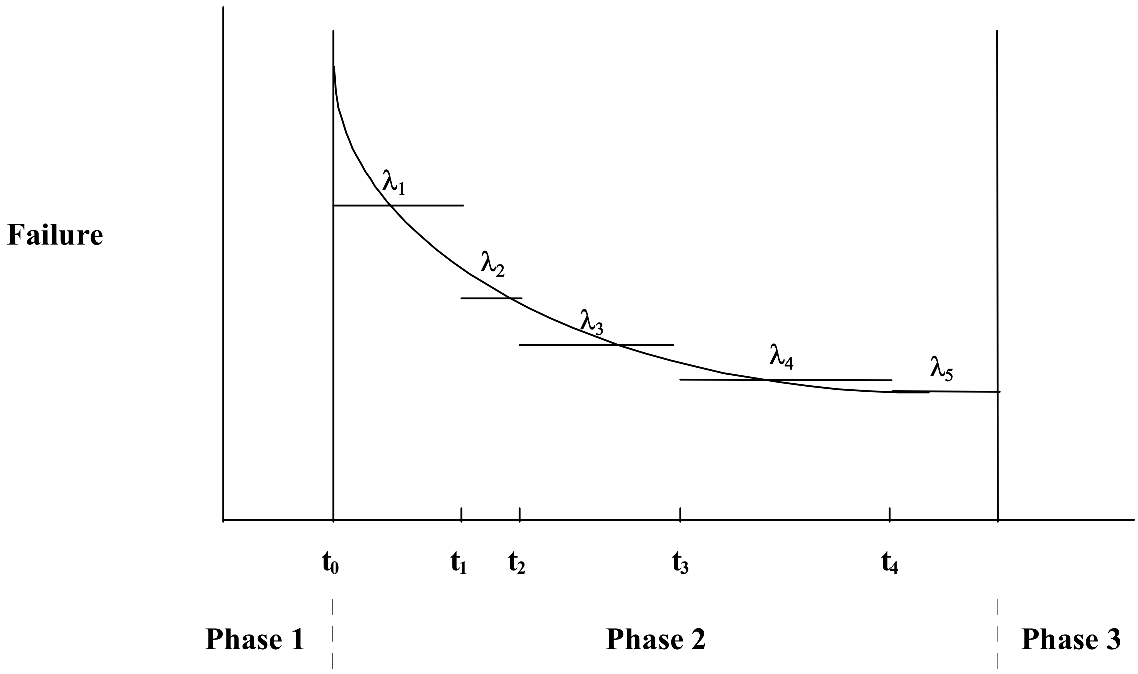

Figure 3.4 Parametric Approximation to Failure Rates Between Modifications

3.2.1.2 The AMSAA Continuous Reliability Growth Tracking Model. The AMSAA Continuous Reliability Growth Tracking Model assumes that within a test phase failures are occurring according to a nonhomogeneous Poisson process with failure rate (intensity of failures) represented by the parametric function:

$$\rho(t) = \lambda \beta t^{\beta-1} \qquad (\lambda, \beta, t > 0) \qquad (12)$$

where the parameter λ is referred to as the scale parameter because it depends upon the unit of measurement chosen for t, the parameter β is referred to as the growth or shape parameter because it characterizes the shape of the graph of the intensity function (equation (12) and Figure 3.4), and t is the cumulative test time. Under this model the function:

$$\mathrm{m(t)} = \frac{1}{\rho(\mathrm{t})} = \left(\lambda \beta t^{\beta-1}\right)^{-1} \qquad (13)$$

is interpreted as the instantaneous mean time between failures (MTBF) of the system at time t. When t corresponds to the total cumulative time for the system; that is, t=T, then m(T) is the demonstrated MTBF of the system in its present configuration at the end of test.

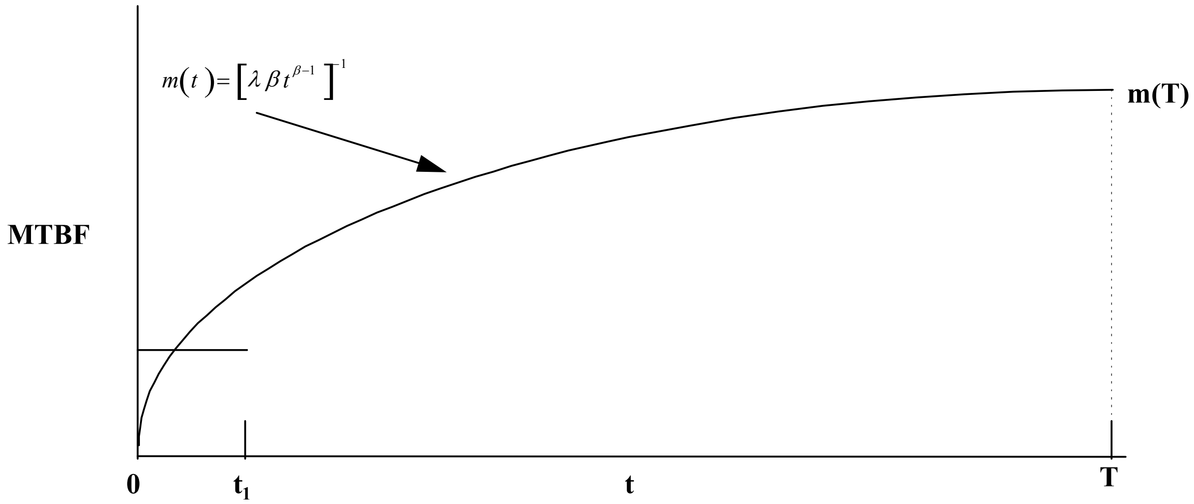

Figure 3.4.1 Test Phase Reliability Growth Based on AMSAA Continuous Tracking Model

Note that the theoretical curve is undefined at the origin. Typically the MTBF during the initial test interval $[0, t_1]$ is characterized by a constant reliability with growth occurring beyond t_1.

Cumulative Number of Failures

The total number of failures F(t) accumulated on all test items in cumulative test time t is a Poisson random variable, and the probability that exactly *f* failures occur between the initiation of testing and the cumulative test time t is:

$$Prob[F(t) = f] \quad = \quad \frac{[\theta(t)]^f \, e^{-\theta(t)}}{f!} \tag{14}$$

in which $\theta(t)$ is the mean value function; that is, the expected number of failures expressed as a function of test time. To describe the reliability growth process, the cumulative number of failures is a function of the form $\theta(t) = \lambda t^{\beta}$, where λ and β are positive parameters.

Number of Failures in an Interval

The number of failures occurring in the interval from test time t_1 until test time t_2, where $t_2 > t_1$ is a Poisson random variable with mean:

$$\theta(t_2) - \theta(t_1) \quad = \quad \lambda\left(t_2^{\beta} - t_1^{\beta}\right)$$

(15)

According to the model assumption, the number of failures that occur in any time interval is statistically independent of the number of failures that occur in any interval which does not overlap the first interval, and only one failure can occur at any instant of time.

Intensity Function

The intensity function in (12) is sometimes referred to as a failure rate; it is not the failure rate of a life distribution, rather it is the failure rate of a process, namely a NHPP.

Option For Individual Failure Time Data

Estimation Procedures For Model

Modeling reliability growth as a nonhomogeneous Poisson process permits an assessment of the demonstrated reliability by statistical procedures. The method of maximum likelihood provides estimates for the scale parameter λ and the shape parameter β, which are used in the estimation of the intensity function $\rho(t)$ in (12). In accordance with (13), the reciprocal of the current value of the intensity function is the instantaneous mean time between failures (MTBF) for the system. Procedures for point estimation and interval estimation for the system MTBF are described in more detail. A goodness-of-fit test to determine model suitability is also described.

The procedures outlined in this section are used to analyze data for which (a) the exact times of failure are known and (b) testing is conducted on a time terminated basis or the tests are in progress with data available through some time. The required data consist of the cumulative test time on all systems at the occurrence of each failure as well as the accumulated total test time T. To calculate the cumulative test time of a failure occurrence, it is necessary to sum the test time on every system at the point of failure. The data then consist of the F successive failure times $X_1 < X_2 < X_3 <\ldots< X_F$ that occur prior to T. This case is referred to as the Option for Individual Failure Time Data.

Point Estimation

The method of maximum likelihood provides point estimates for the parameters of the failure intensity function (12). The maximum likelihood estimate (mle) for the shape parameter β is:

$$\hat{\beta} = \frac{F}{F \ln T - \sum_{i=1}^{F} \ln X_i} \tag{16}$$

By equating the observed number of failures by time T (namely F) with the expected number of failures by time T (namely E[F(T)]) and by substituting mle's in place of the true, but unknown, parameters in (10) we obtain:

$$\mathrm{F} = \hat{\lambda}\,\mathrm{T}^{\hat{\beta}} \tag{17}$$

from which we obtain an estimate for the scale parameter λ:

$$\hat{\lambda} = \frac{\mathrm{F}}{\mathrm{T}^{\hat{\beta}}} \tag{18}$$

For any time t > 0, the failure intensity function is estimated by:

$$\hat{\rho}(t) = \hat{\lambda}\hat{\beta}\, t^{\hat{\beta}-1} \qquad (19)$$

In particular, (19) holds for the total test time T. By substitution from (17), the estimator $\hat{\rho}(T)$ can be written as:

$$\hat{\rho}(T) = \hat{\lambda}\,\hat{\beta}\, T^{\hat{\beta}-1} = \hat{\beta}\left(\frac{\hat{\lambda}\ T^{\hat{\beta}}}{T}\right) = \hat{\beta}\left(\frac{F}{T}\right) \qquad (20)$$

where F/T is the estimate of the intensity function for a homogeneous Poisson process. Hence the fraction $(1-\hat{\beta})$ of the initial failure intensity is effectively removed by time T, resulting in (20).

Finally, the reciprocal of $\hat{\rho}(T)$ provides an estimate of the mean time between failures of the system at the time T and represents the system reliability growth under the model:

$$\hat{m}(T) = \frac{1}{\hat{\rho}(T)} = \left(\hat{\lambda}\hat{\beta}\, T^{\hat{\beta}-1}\right)^{-1} \qquad (21)$$

Interval Estimation

Interval estimates provide a measure of the uncertainty regarding a parameter. For the reliability growth process, the parameter of primary interest is the system mean time between failures at the end of test, m(T). The probability distribution of the point estimate for the intensity function at T, $\hat{\rho}(T)$, is the basis for the interval estimate for the true (but unknown) value of the intensity function at T, $\rho(T)$.

These interval estimates are referred to as confidence intervals and may be computed for selected confidence levels. The values in Table C-1 facilitate computation of two-sided confidence intervals for m(T) by providing confidence coefficients L and U corresponding to the lower bound and upper bound, respectively. These coefficients are indexed by the total number of observed failures F and the desired confidence level γ. The two-sided confidence interval for m(T) is thus:

$$L_{F,\gamma}\hat{m}(T) \le m(T) \le U_{F,\gamma}\hat{m}(T) \qquad (22)$$

Table C-2 may be used to compute one-sided interval estimates (lower confidence bounds) for m(T) such that:

$$L_{F,\gamma}\hat{m}(T) \le m(T) \qquad (23)$$

Note that both tables are to be used only for time terminated growth tests. Also, since the number of failures has a discrete probability distribution, the interval estimates in (22) and (23) are conservative; that is, the actual confidence level is slightly larger than the desired confidence level γ.

TABLE C-1. LOWER (L) AND UPPER (U) COEFFICIENTS FOR CONFIDENCE INTERVALS FOR MTBF FROM TIME TERMINATED RELIABILITY GROWTH TEST

γ / F	.80 L	.80 U	.90 L	.90 U	.95 L	.95 U	.98 L	.98 U
2	.261	18.66	.200	38.66	.159	78.66	.124	198.7
3	.333	6.326	.263	9.736	.217	14.55	.174	24.10
4	.385	4.243	.312	5.947	.262	8.093	.215	11.81
5	.426	3.386	.352	4.517	.300	5.862	.250	8.043
6	.459	2.915	.385	3.764	.331	4.738	.280	6.254
7	.487	2.616	.412	3.298	.358	4.061	.305	5.216
8	.511	2.407	.436	2.981	.382	3.609	.328	4.539
9	.531	2.254	.457	2.750	.403	3.285	.349	4.064
10	.549	2.136	.476	2.575	.421	3.042	.367	3.712
11	.565	2.041	.492	2.436	.438	2.852	.384	3.441
12	.579	1.965	.507	2.324	.453	2.699	.399	3.226
13	.592	1.901	.521	2.232	.467	2.574	.413	3.050
14	.604	1.846	.533	2.153	.480	2.469	.426	2.904
15	.614	1.800	.545	2.087	.492	2.379	.438	2.781
16	.624	1.759	.556	2.029	.503	2.302	.449	2.675
17	.633	1.723	.565	1.978	.513	2.235	.460	2.584
18	.642	1.692	.575	1.933	.523	2.176	.470	2.503
19	.650	1.663	.583	1.893	.532	2.123	.479	2.432
20	.657	1.638	.591	1.858	.540	2.076	.488	2.369
21	.664	1.615	.599	1.825	.548	2.034	.496	2.313
22	.670	1.594	.606	1.796	.556	1.996	.504	2.261
23	.676	1.574	.613	1.769	.563	1.961	.511	2.215
24	.682	1.557	.619	1.745	.570	1.929	.518	2.173
25	.687	1.540	.625	1.722	.576	1.900	.525	2.134
26	.692	1.525	.631	1.701	.582	1.873	.531	2.098
27	.697	1.511	.636	1.682	.588	1.848	.537	2.068
28	.702	1.498	.641	1.664	.594	1.825	.543	2.035
29	.706	1.486	.646	1.647	.599	1.803	.549	2.006
30	.711	1.475	.651	1.631	.604	1.783	.554	1.980
35	.729	1.427	.672	1.565	.627	1.699	.579	1.870
40	.745	1.390	.690	1.515	.646	1.635	.599	1.788
45	.758	1.361	.705	1.476	.662	1.585	.617	1.723
50	.769	1.337	.718	1.443	.676	1.544	.632	1.671
60	.787	1.300	.739	1.393	.700	1.481	.657	1.591
70	.801	1.272	.756	1.356	.718	1.435	.678	1.533
80	.813	1.251	.769	1.328	.734	1.399	.695	1.488
100	.831	1.219	.791	1.286	.758	1.347	.722	1.423

γ = confidence level

For $F > 100$, $\quad L \approx \left(1 + {z_{.5+\gamma/2}}\big/{\sqrt{2F}}\right)^{-2}$ *and* $U \approx \left(1 - {z_{.5+\gamma/2}}\big/{\sqrt{2F}}\right)^{-2}$

in which $z_{.5+\gamma/2}$ is the $100 \times \left(.5 + \gamma/2\right) - th$ percentile of the standard normal distribution.

TABLE C-2. LOWER CONFIDENCE INTERVAL COEFFICIENTS FOR MTBF FROM TIME TERMINATED RELIABILITY GROWTH TEST

	Confidence Level γ								Confidence Level γ						
F	.50	.60	.70	.80	.90	.95	.99	F	.50	.60	.70	.80	.90	.95	.99
2	.761	.606	.480	.369	.261	.200	.124	51	.987	.939	.891	.838	.771	.720	.635
3	.823	.680	.559	.447	.333	.263	.174	52	.987	.940	.892	.840	.773	.722	.637
4	.860	.727	.611	.501	.385	.312	.215	53	.988	.940	.893	.841	.775	.724	.640
5	.884	.760	.649	.542	.426	.352	.250	54	.988	.941	.894	.843	.777	.727	.643
6	.901	.784	.678	.574	.459	.385	.280	55	.988	.942	.895	.844	.778	.729	.645
7	.914	.803	.701	.600	.487	.412	.305	56	.988	.942	.896	.845	.780	.731	.648
8	.924	.818	.720	.622	.511	.436	.328	57	.988	.943	.897	.847	.782	.733	.650
9	.932	.830	.736	.640	.531	.457	.349	58	.989	.944	.898	.848	.784	.735	.653
10	.938	.841	.749	.656	.549	.476	.367	59	.989	.944	.899	.849	.785	.737	.655
11	.943	.849	.761	.670	.565	.492	.384	60	.989	.945	.900	.850	.787	.739	.657
12	.948	.857	.771	.683	.579	.507	.399	61	.989	.945	.901	.852	.788	.741	.659
13	.951	.864	.780	.694	.592	.521	.413	62	.989	.946	.901	.853	.790	.742	.662
14	.955	.870	.788	.704	.604	.533	.426	63	.990	.946	.902	.854	.792	.744	.664
15	.958	.875	.795	.713	.614	.545	.438	64	.990	.947	.903	.855	.793	.746	.666
16	.960	.880	.802	.721	.624	.556	.449	65	.990	.947	.904	.856	.794	.748	.668
17	.962	.884	.808	.729	.633	.565	.460	66	.990	.948	.905	.857	.796	.749	.670
18	.964	.888	.814	.736	.642	.575	.470	67	.990	.948	.905	.858	.797	.751	.672
19	.966	.891	.819	.742	.650	.583	.479	68	.990	.948	.906	.859	.799	.752	.674
20	.968	.895	.823	.748	.657	.591	.488	69	.990	.949	.907	.860	.800	.754	.676
21	.969	.898	.828	.754	.664	.599	.496	70	.991	.949	.907	.861	.801	.756	.678
22	.971	.900	.832	.759	.670	.606	.504	71	.991	.950	.908	.862	.803	.757	.680
23	.972	.903	.836	.764	.676	.613	.511	72	.991	.950	.909	.863	.804	.759	.681
24	.973	.905	.839	.769	.682	.619	.518	73	.991	.951	.909	.864	.805	.760	.683
25	.974	.908	.842	.773	.687	.625	.525	74	.991	.951	.910	.865	.806	.761	.685
26	.975	.910	.846	.777	.692	.631	.531	75	.991	.951	.911	.866	.807	.763	.687
27	.976	.912	.849	.781	.697	.636	.537	76	.991	.952	.911	.866	.809	.764	.688
28	.977	.914	.851	.785	.702	.641	.543	77	.991	.952	.912	.867	.810	.766	.690
29	.978	.915	.854	.788	.706	.646	.549	78	.992	.952	.912	.868	.811	.767	.692
30	.978	.917	.857	.792	.711	.651	.554	79	.992	.953	.913	.869	.812	.768	.693
31	.979	.919	.859	.795	.715	.656	.560	80	.992	.953	.914	.870	.813	.769	.695
32	.980	.920	.861	.798	.719	.660	.565	81	.992	.953	.914	.871	.814	.771	.696
33	.980	.922	.863	.801	.722	.664	.570	82	.992	.954	.915	.871	.815	.772	.698
34	.981	.923	.866	.804	.726	.668	.574	83	.992	.954	.915	.872	.816	.773	.699
35	.981	.924	.868	.806	.729	.672	.579	84	.992	.854	.916	.873	.817	.774	.701
36	.982	.926	.869	.809	.733	.676	.583	85	.992	.955	.916	.874	.818	.776	.702
37	.982	.927	.871	.811	.736	.680	.587	86	.992	.955	.917	.874	.819	.777	.704
38	.983	.928	.873	.814	.739	.683	.591	87	.992	.955	.917	.875	.820	.778	.705
39	.983	.929	.875	.816	.742	.687	.595	88	.992	.956	.918	.876	.821	.779	.707
40	.984	.930	.876	.818	.745	.690	.599	89	.993	.956	.918	.876	.822	.780	.708
41	.984	.931	.878	.820	.747	.693	.603	90	.993	.956	.919	.877	.823	.781	.709
42	.984	.932	.879	.822	.750	.696	.606	91	.993	.956	.919	.878	.824	.782	.711
43	.985	.933	.881	.824	.753	.699	.610	92	.993	.957	.920	.878	.825	.783	.712
44	.985	.934	.882	.826	.755	.702	.613	93	.993	.957	.920	.879	.826	.784	.713
45	.985	.935	.884	.828	.758	.705	.617	94	.993	.957	.920	.880	.826	.785	.714
46	.986	.935	.885	.830	.760	.707	.620	95	.993	.957	.921	.880	.827	.786	.716
47	.986	.936	.886	.832	.762	.710	.623	96	.993	.958	.921	.881	.828	.787	.717
48	.986	.937	.888	.833	.764	.713	.626	97	.993	.958	.922	.881	.829	.788	.718
49	.987	.938	.889	.835	.767	.715	.629	98	.993	.958	.922	.882	.830	.789	.719
50	.987	.939	.890	.837	.769	.718	.632	99	.993	.958	.923	.883	.831	.790	.721

Goodness-of-Fit

For the case where the individual failure times are known, a Cramér-von Mises statistic is used to test the null hypothesis that a nonhomogeneous Poisson process with failure intensity function (12) properly describes the reliability growth of a system. To calculate the statistic, an unbiased estimate of the shape parameter β is used:

$$\bar{\beta} = \frac{F-1}{F}\hat{\beta} \tag{24}$$

This unbiased estimate of β is for a time terminated reliability growth test with F observed failures. The goodness-of-fit statistic is:

$$C_F = \frac{1}{12F} + \sum_{i=1}^{F}\left[\left(\frac{X_i}{T}\right)^{\bar{\beta}} - \frac{2i-1}{2F}\right]^2 \tag{25}$$

where the failure times X_i must be ordered so that $0 < X_1 \le X_2 \le \ldots < X_F$.

The null hypothesis that the model represents the observed data is rejected if the statistic C_F exceeds the critical value for a chosen significance level α. Critical values of C_F for α = .20, .15, .10, .05, .01 are shown in Table C-3 where the table is indexed by F, the total number of observed failures.

TABLE C-3. CRITICAL VALUES FOR CRAMÉR-VON MISES GOODNESS-OF-FIT TEST FOR INDIVIDUAL FAILURE TIME DATA

F \ α	.20	.15	.10	.05	.01
2	.138	.149	.162	.175	.186
3	.121	.135	.154	.184	.23
4	.121	.134	.155	.191	.28
5	.121	.137	.160	.199	.30
6	.123	.139	.162	.204	.31
7	.124	.140	.165	.208	.32
8	.124	.141	.165	.210	.32
9	.125	.142	.167	.212	.32
10	.125	.142	.167	.212	.32
11	.126	.143	.169	.214	.32
12	.126	.144	.169	.214	.32
13	.126	.144	.169	.214	.33
14	.126	.144	.169	.214	.33
15	.126	.144	.169	.215	.33
16	.127	.145	.171	.216	.33
17	.127	.145	.171	.217	.33
18	.127	.146	.171	.217	.33
19	.127	.146	.171	.217	.33
20	.128	.146	.172	.217	.33
30	.128	.146	.172	.218	.33
60	.128	.147	.173	.220	.33
100	.129	.147	.173	.220	.34

For F > 100 use values for F = 100.
α = significance level

Besides using statistical methods for assessing model goodness-of-fit, one should also construct an average failure rate plot or a superimposed expected failure rate plot (as shown in Figure 3.4.2). These plots, derived from the failure data, provide a graphic description of test results and should always be part of the reliability analysis.

Example

The following example demonstrates the option for individual failure time data in which two prototypes of a system are tested concurrently with the incorporation of design changes. (The data in this example are used subsequently for one of the growth subsystems in the example for the AMSAA Subsystem Tracking Model - SSTRACK.) The first prototype is tested for 132.4 hours, and the second is tested for 167.6 hours for a total of T = 300 cumulative test hours. Table C-4 shows the time on each prototype and the cumulative test time at each failure occurrence. An asterisk denotes the failed system. There are a total of F = 27 failures. Although the occurrence of two failures at exactly 16.5 hours is not possible under the assumption of the model, such data can result from rounding and are computationally tractable using the statistical estimation procedures described previously for the model. Note that the data are from a time terminated test.

TABLE C-4. TEST DATA FOR INDIVIDUAL FAILURE TIME OPTION
(An asterisk denotes the failed system.)

Failure Number	Prot. #1 Hours	Prot. #2 Hours	Cum Hours	Failure Number	Prot. #1 Hours	Prot. #2 Hours	Cumulative Hours
1	2.6*	.0	2.6	15	60.5	37.6*	98.1
2	16.5*	.0	16.5	16	61.9*	39.1	101.1
3	16.5*	.0	16.5	17	76.6*	55.4	132.0
4	17.0*	.0	17.0	18	81.1	61.1*	142.2
5	20.5	.9*	21.4	19	84.1*	63.6	147.7
6	25.3	3.8*	29.1	20	84.7*	64.3	149.0
7	28.7	4.6*	33.3	21	94.6*	72.6	167.2
8	41.8*	14.7	56.5	22	104.8	85.9*	190.7
9	45.5*	17.6	63.1	23	105.9	87.1*	193.0
10	48.6	22.0*	70.6	24	108.8*	89.9	198.7
11	49.6	23.4*	73.0	25	132.4	119.5*	251.9
12	51.4*	26.3	77.7	26	132.4	150.1*	282.5
13	58.2*	35.7	93.9	27	132.4	153.7*	286.1
14	59.0	36.5*	95.5	End	132.4	167.6	300.0

By using the 27 failure times listed under the columns labeled "Cumulative Hours" in Table C-4 and by applying (16), (18), (19) and (21) we obtain the following estimates. The point estimate for the shape parameter is $\hat{\beta} = 0.716$; the point estimate for the scale parameter is $\hat{\lambda} = 0.454$; the estimated failure intensity at the end of the test is $\hat{\rho}(\mathrm{T}) = 0.0645$ failures per hour; the estimated MTBF at the end of the 300-hour test is $\hat{\mathrm{m}}(\mathrm{T}) = 15.5$ hours. As shown in Figure 3.4.2, superimposing a graph of the estimated intensity function [19] atop a plot of the average failure rate (using six 50-hour intervals) reveals a decreasing failure intensity indicative of reliability growth.

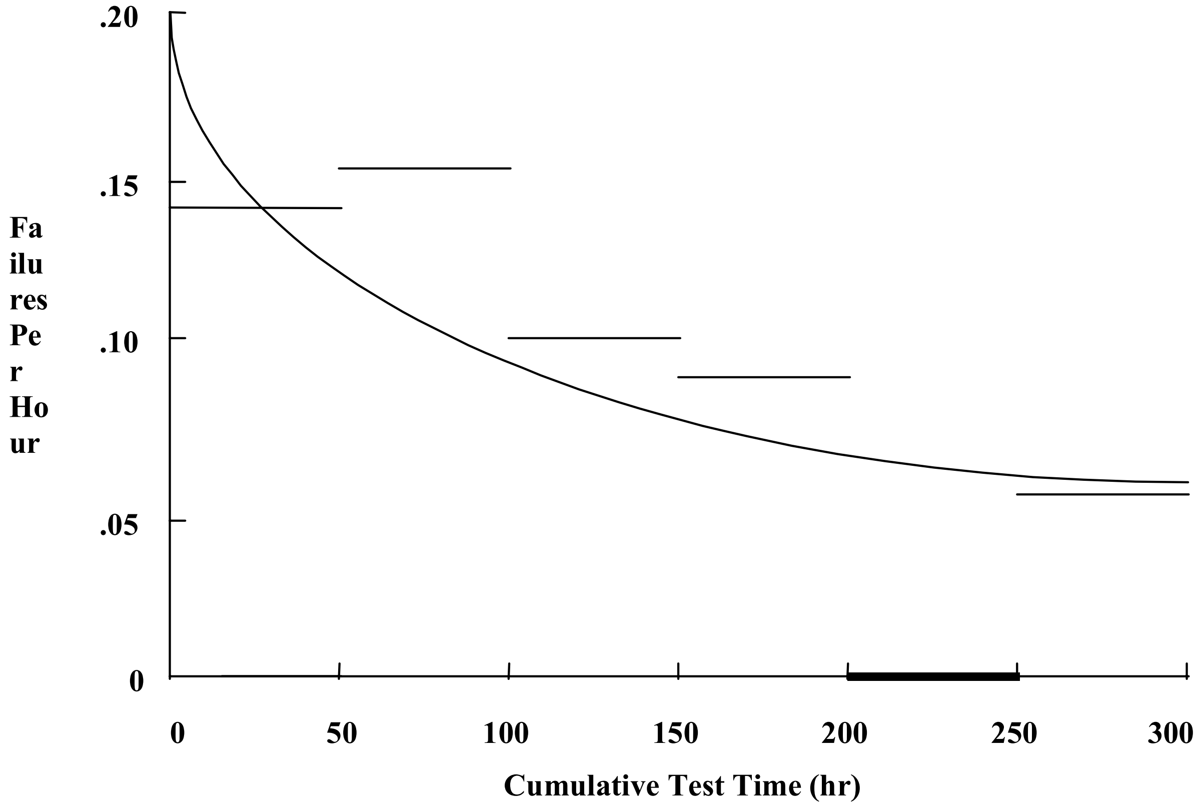

Figure 3.4.2 Estimated Intensity Function Superimposed On Average Failure Rate Plot From Observed Data

Using (22), Table C-1 and a confidence level of 90 percent, the two-sided interval estimate for the MTBF at the end of the test is [9.9, 26.1]. These results and the estimated MTBF tracking growth curve (substituting t for T in (21)) are shown in Figure 3.4.3.

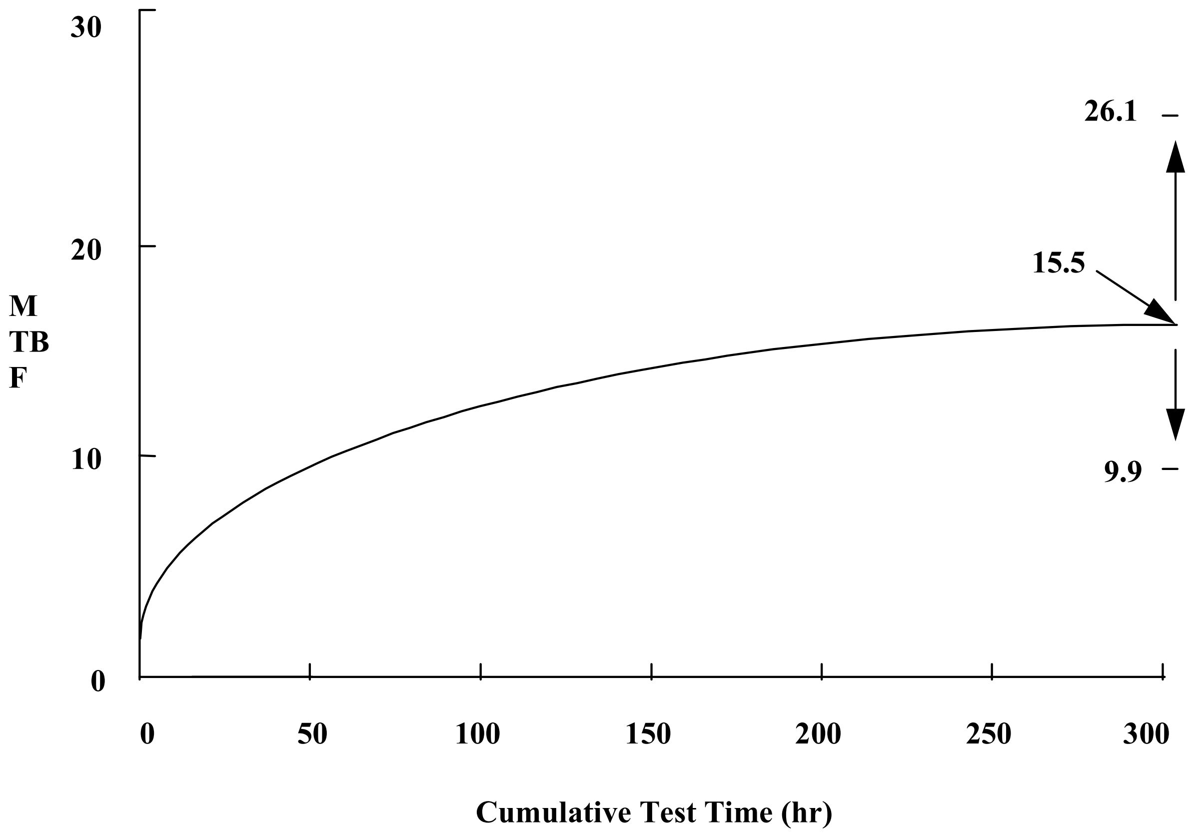

Figure 3.4.3 Estimated MTBF Function With 90 Percent Interval Estimate at T = 300 Hours

Finally, to test the model goodness-of-fit, a Cramér-von Mises statistic is compared to the critical value from Table C-3 corresponding to a chosen significance level $\alpha = 0.05$ and total observed number of failures F = 27. Linear interpolation is used to arrive at the critical value. Since the statistic, 0.091, is less than the critical value, 0.218, we accept the hypothesis that the AMSAA Continuous Reliability Growth Tracking Model is appropriate for this data set.

Option for Grouped Data

List of Notation

K	number of intervals (or groups) or the last group
i	interval number
t_i	time at beginning (or end) of interval
F_i	observed number of failures in interval $[t_{i-1}, t_i)$
t_K	total test time
^	denotes an estimate when placed over a parameter
β	shape parameter $(\beta > 0)$

λ	scale parameter $(\lambda > 0)$
$\rho(t)$	instantaneous failure intensity at time t
$m(t)$	instantaneous MTBF at time t
M_K	MTBF for the last group
E_K	expected number of failures in the last group
ρ_K	failure intensity for the last group
F	total observed number of failures
L	lower confidence coefficient
U	upper confidence coefficient
γ	specified confidence level
E_i	expected number of failures in interval i
K_R	number of intervals after recombination of intervals
O_i	observed number of failures in interval i
χ^2	chi-squared value

Reliability growth parameters can be estimated in accordance with the AMSAA Continuous Tracking Model even if the exact times of failure are unknown and all that is known is the number of failures that occurred in each interval of time, provided there are at least three intervals and at least two intervals have failures. This case is referred to as the Option for Grouped Data. This section describes the estimation procedures and goodness-of-fit procedures for analyzing such data and provides an example of model usage. In the following discussion, the words "group" and "interval" are interchangeable.

Estimation Procedures for Model

The required data consist of the total number of failures in each of K intervals of test time. The first interval always starts at test time zero so that $t_0 = 0$. The groups do not have to be of equal length. The observed number of failures in the interval from t_{i-1} to t_i is denoted by F_i.

Point Estimation

The method of maximum likelihood provides point estimates for the parameters of the model. The maximum likelihood estimate for the shape parameter β is the value that satisfies the following nonlinear equation:

$$\sum_{i=1}^{K} F_i \left[\frac{t_i^{\hat{\beta}} \ln t_i - t_{i-1}^{\hat{\beta}} \ln t_{i-1}}{t_i^{\hat{\beta}} - t_{i-1}^{\hat{\beta}}} - \ln t_K \right] = 0 \qquad (26)$$

in which $t_0 \ln t_0$ is defined as zero.

By equating the total expected number of failures to the total observed number of failures:

$$\hat{\lambda}\, t_K^{\hat{\beta}} = \sum_{i=1}^{K} F_i \tag{27}$$

and solving for $\hat{\lambda}$, we obtain an estimate for the scale parameter:

$$\hat{\lambda} = \frac{\sum_{i=1}^{K} F_i}{t_K^{\hat{\beta}}} \tag{28}$$

Point estimates for the intensity function $\rho(t)$ and the mean time between failures function $m(t)$ are calculated as in the previous section describing the Option for Individual Failure Time Data; that is,

$$\hat{\rho}(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1} \qquad \left(\hat{\lambda}, \hat{\beta}, t > 0\right) \tag{29}$$

$$\hat{m}(t) = \hat{\rho}(t)^{-1} \qquad \left(\hat{\lambda}, \hat{\beta}, t > 0\right) \tag{30}$$

The functions in (29) and (30) provide instantaneous estimates that give rise to smooth continuous curves, but these functions do not describe the reliability growth that occurs on a configuration basis representative of grouped data. Under the model option for grouped data, the estimate for the MTBF for the last group, $\hat{M}_K$, is the amount of test time in the last group divided by the expected number of failures in the last group:

$$\hat{M}_K = \frac{t_K - t_{K-1}}{E_K} \tag{31}$$

where the expected number of failures in the last group E_K is:

$$E_K = \hat{\lambda}\left(t_K^{\hat{\beta}} - t_{K-1}^{\hat{\beta}}\right) \tag{32}$$

From (31) we obtain an estimate for the failure intensity for the last group:

$$\hat{\rho}_K = \frac{1}{\hat{M}_K} \tag{33}$$

Interval Estimation

Lower confidence bounds and two-sided confidence intervals may be computed for the MTBF for the last group. Using (31) and Table C-1, a two-sided confidence interval for M_K may be calculated from:

$$L_{F,\gamma}\,\hat{M}_K \quad \le \quad M_K \;\le\; U_{F,\gamma}\,\hat{M}_K \tag{34}$$

and using (31) and Table C-2, a one-sided interval estimate for M_K may be calculated from:

$$L_{F,\gamma}\,\hat{M}_K \quad \le \quad M_K \tag{35}$$

where F is the total observed number of failures and γ is the desired confidence level.

Goodness-of-Fit

A chi-squared goodness-of-fit test is used to test the null hypothesis that the AMSAA Continuous Reliability Growth Tracking Model adequately represents a set of grouped data. The expected number of failures in the interval from t_{i-1} to t_i is approximated by:

$$E_i \quad = \quad \hat{\lambda}\left(t_i^{\hat{\beta}} - t_{i-1}^{\hat{\beta}}\right) \tag{36}$$

Adjacent intervals may have to be combined so that the expected number of failures in any combined interval is at least five. Let the number of intervals after this recombination be K_R, and let the observed number of failures in the i-th new interval be O_i and the expected number of failures in the i-th new interval be E_i. Then the statistic:

$$\chi^2 \quad = \quad \sum_{i=1}^{K_R} \frac{\left(O_i - E_i\right)^2}{E_i} \tag{37}$$

is approximately distributed as a chi-squared random variable with $K_R - 2$ degrees of freedom. The null hypothesis is rejected if the χ^2 statistic exceeds the critical value for a chosen significance level. Critical values for this statistic can be found in tables of the chi-squared distribution.

Besides using statistical methods for assessing model goodness-of-fit, one should also construct an average failure rate plot or a superimposed expected failure rate plot (as shown in Figure 3.4.2). Derived from the failure data, these plots provide a graphic description of test results and should always be part of the reliability analysis.

Example

The following example uses aircraft data to demonstrate the option for grouped data. (The data in this example are used subsequently for one of the growth subsystems in the example for the AMSAA Subsystem Tracking Model - SSTRACK.) In this example, an aircraft has scheduled inspections at intervals of twenty flight hours. For the first 100 hours of flight testing the results are:

TABLE C-5. TEST DATA FOR GROUPED DATA OPTION

Start Time	End Time	Observed Number of Failures
0	20	13
20	40	16
40	60	5
60	80	8
80	100	7

There are a total of F = 49 observed failures from K = 5 intervals. Solution of (26) for $\hat{\beta}$ yields an estimate of 0.753 for the shape parameter. From (28) the scale parameter estimate is 1.53. For the last group, the intensity function estimate is 0.379 failures per flight hour and the MTBF estimate is 2.6 flight hours. Table C-6 shows that those adjacent intervals do not have to be combined after applying (36) to the original intervals. Therefore, $K_R = 5$.

TABLE C-6. OBSERVED VERSUS EXPECTED NUMBER OF FAILURES FOR TEST DATA FOR GROUPED DATA OPTION

Start Time	End Time	Observed Number of Failures	Expected Number of Failures
0	20	13	14.59
20	40	16	9.99
40	60	5	8.77
60	80	8	8.07
80	100	7	7.58

To test the model goodness-of-fit, a chi-squared statistic of 5.5 is compared to the critical value of 7.8 corresponding to 3 degrees of freedom and a 0.05 significance level. Since the statistic is less than the critical value, the applicability of the model is accepted.

3.2.2 AMSAA Discrete Tracking Model.

3.2.2.1 Background and Motivation for Model.

List of Notation

t	cumulative test time
K(t)	cumulative number of failures by time t
c(t)	cumulative failure rate by time t
ln	natural logarithm function (base e)
δ	constant term representing the y-intercept of a linear equation
α	constant term representing the slope of a linear equation
λ	scale parameter ($\lambda > 0$) of power function

β	shape parameter ($\beta > 0$) of power function; $\beta = 1 - \alpha$
i	configuration number
T_i	cumulative number of trials through configuration i
Σ	summation of
N_i	number of trials in configuration i
K_i	cumulative number of failures through configuration i
M_i	number of failures in configuration i
$E[K_i]$	expected value of K_i
f_i	probability of failure for configuration i
g_i	probability of failure for trial i
R_i	reliability for configuration i (or trial i)
^	denotes an estimate when placed over a parameter

Reliability growth tracking methodology may also be applied to discrete data in a manner that is consistent with the learning curve property observed by J.T. Duane for continuous data. Accordingly, this section describes model development and maximum likelihood estimation procedures for assessing system reliability for one-shot systems during development.

The motivation for the AMSAA Discrete Reliability Growth Tracking Model comes from the learning curve approach for continuous data as follows.

Let t denote the cumulative test time, and let K(t) denote the cumulative number of failures by time t. The cumulative failure rate, c(t), is the ratio:

$$c(t) = \frac{K(t)}{t} \tag{38}$$

While plotting test data from generators, hydro-mechanical devices and aircraft jet engines, Duane observed that the logarithm of the cumulative failure rate was linear when plotted against the logarithm of the cumulative test time:

$$\ln c(t) = \delta - \alpha \ln t \tag{39}$$

By letting $\delta = \ln \lambda$ for the y-intercept and by exponentiating both sides of (39), the cumulative failure rate becomes:

$$c(t) = \lambda\, t^{-\alpha} \tag{40}$$

By substitution from (38),

$$\frac{K(t)}{t} = \lambda\, t^{-\alpha} \tag{41}$$

Multiplying both sides of (41) by t and letting $\beta = 1-\alpha$, the cumulative number of failures by t becomes:

$$K(t) = \lambda t^{\beta} \tag{42}$$

This power function of t is the learning curve property for K(t), where $\lambda, \beta > 0$.

3.2.2.2 Model Development. To construct the AMSAA Discrete Reliability Growth Tracking Model, we use the power function developed from the learning curve property for K(t) to derive an equation for the probability of failure on a configuration basis. We refer to this situation where growth takes place on a configuration basis (and the number of trials in at least one of the configurations is greater than one) as the grouped data option. In the presence of reliability growth, the failure probability trend for the grouped data option appears graphically as a sequence of decreasing, horizontal steps.

We then note the special case where the configuration size is one for all configurations, develop an equation for the probability of failure, and refer to this special case as the option for trial by trial data. In a growth situation, the failure probability trend for this option is described graphically as a decreasing, smooth curve.

Model development proceeds as follows. Suppose system development is represented by i configurations. (This corresponds to $i-1$ configuration changes, unless fixes are applied at the end of the test phase, in which case there would be i configuration changes.) Let N_i be the number of trials during configuration i, and let M_i be the number of failures during configuration i. Then the cumulative number of trials through configuration i, namely T_i, is the sum of the N_i for all i:

$$T_i = \sum N_i \tag{43}$$

and the cumulative number of failures through configuration i, namely K_i, is the sum of the M_i for all i:

$$K_i = \sum M_i \tag{44}$$

We express the expected value of K_i as $E[K_i]$ and define it as the expected number of failures by the end of configuration i. Applying the learning curve property to $E[K_i]$ implies:

$$E[K_i] = \lambda T_i^{\beta} \tag{45}$$

We introduce a term for the probability of failure for configuration one, namely f_1, and use it to develop a generalized equation for f_i in terms of the T_i and N_i. From (45), the expected number of failures by the end of configuration one is:

$$E[K_1] = \lambda T_1^{\beta} = f_1 N_1 \Rightarrow f_1 = \frac{\lambda T_1^{\beta}}{N_1} \tag{46}$$

Applying (45) again and noting that the expected number of failures by the end of configuration two is the sum of the expected number of failures in configuration one and the expected number of failures in configuration two, we obtain:

$$E[K_2] = \lambda T_2^{\beta} = f_1 N_1 + f_2 N_2 = \lambda T_1^{\beta} + f_2 N_2 \Rightarrow f_2 = \frac{\lambda T_2^{\beta} - \lambda T_1^{\beta}}{N_2} \tag{47}$$

By this method of inductive reasoning we obtain a generalized equation for the failure probability, f_i, on a configuration basis:

$$f_i = \frac{\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}}{N_i} \tag{48}$$

and use (48) for the grouped data option.

For the special case where $N_i = 1$ for all i, (48) becomes a smooth curve, g_i, that represents the probability of failure for the option for trial by trial data:

$$g_i = \lambda i^{\beta} - \lambda (i-1)^{\beta} \tag{49}$$

In (49), i represents the trial number. Note that $T_0 = 0$, so that (48) reduces to (46) when $i = 1$. Also, for $i = 1$ in (49), $g_1 = \lambda$. Using (48) we obtain an equation for the reliability (probability of success) for the i-th configuration:

$$R_i = 1 - f_i \tag{50}$$

and using (49) we obtain an equation for the reliability for the i-th trial:

$$R_i = 1 - g_i \tag{51}$$

Equations (48), (49), (50) and (51) are the exact model equations for tracking the reliability growth of discrete data using the AMSAA Discrete Reliability Growth Tracking Model.

3.2.2.3 Estimation Procedures. This section describes procedures for estimating the parameters of the AMSAA Discrete Reliability Growth Tracking Model. It also includes an approximation equation for calculating reliability lower confidence bounds and an example illustrating these concepts.

The estimation procedures described below provide maximum likelihood estimates (mle's) for the model's two parameters, λ and β, where λ is the scale parameter and β is the shape (or growth) parameter. The mle's for λ and β allow for point estimates for the probability of failure:

$$\hat{f}_i = \frac{\hat{\lambda}T_i^{\hat{\beta}} - \hat{\lambda}T_{i-1}^{\hat{\beta}}}{N_i} = \frac{\hat{\lambda}\left(T_i^{\hat{\beta}} - T_{i-1}^{\hat{\beta}}\right)}{N_i} \tag{52}$$

and the probability of success (reliability):

$$\hat{R}_i = 1 - \hat{f}_i \tag{53}$$

for each configuration i.

Point Estimation

Exact mle's for λ and β are values satisfying the following two equations:

$$\sum_{i=1}^{K}\left[\lambda T_i^{\beta}\ln \mathrm{T}_i - \lambda T_{i-1}^{\beta}\ln T_{i-1}\right]\left\{\frac{M_i}{\left[\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}\right]} - \frac{N_i - M_i}{\left[N_i - \lambda T_i^{\beta} + \lambda T_{i-1}^{\beta}\right]}\right\} = 0 \tag{54}$$

and

$$\sum_{i=1}^{K}\left[T_i^{\beta} - T_{i-1}^{\beta}\right]\left\{\frac{M_i}{\left[\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}\right]} - \frac{N_i - M_i}{\left[N_i - \lambda T_i^{\beta} + \lambda T_{i-1}^{\beta}\right]}\right\} = 0 \tag{55}$$

From (54) and (55) we note the following data requirements for using the model:

Data Requirements

K	number of configurations (or the final configuration)
M_i	number of observed failures for configuration i
N_i	number of trials for configuration i
T_i	cumulative number of trials through configuration i

Interval Estimation

A one-sided interval estimate (lower confidence bound) for the reliability of the final (last) configuration may be obtained from the approximation equation:

$$LCB_{\gamma} \approx 1 - \left(1 - \hat{R}_K\right)\left(\frac{\chi^2_{\gamma,n+2}}{n}\right) \tag{56}$$

where

LCB_γ = a lower confidence bound at the gamma (γ) confidence level for the reliability of the last configuration, where γ is a decimal number in the interval (0,1)

$\hat{R}_K$ = a maximum likelihood estimate for the reliability of the last configuration

n = the total number of observed failures (summed) over all configurations i, (i = 1..K)

$\chi^2_{\gamma,n+2}$ = the gamma percentile point of the chi-squared distribution with n+2 degrees of freedom

3.2.2.4 Goodness-of-Fit. Provided there is sufficient data to obtain at least five expected number of failures per group, a chi-squared goodness-of-fit test may be used to test the null hypothesis that the AMSAA Discrete Reliability Growth Tracking Model adequately represents a set of grouped discrete data or a set of trial by trial data. If these conditions are met, then one may use the chi-squared goodness-of-fit procedures outlined previously for the Continuous Reliability Growth Tracking Model.

Besides using statistical methods for assessing model goodness-of-fit, one should also construct an average failure rate plot or a superimposed expected failure rate plot (as shown in Figure 3.4.2). Derived from the failure data, these plots provide a graphic description of test results and should always be part of the reliability analysis.

3.2.2.5 Example. The following example is an application of the grouped data option of the AMSAA Discrete Reliability Growth Tracking Model for a system having four configurations of development test data:

TABLE C-7. TEST DATA FOR GROUPED DATA OPTION

Configuration Number, I K = 4	Observed Number of Failures in Configuration i M_i	Number of Trials in Configuration i N_i	Cumulative Number of Trials Through Configuration i T_i
1	5	14	14
2	3	19	33
3	4	15	48
4	4	20	68

This is represented graphically as:

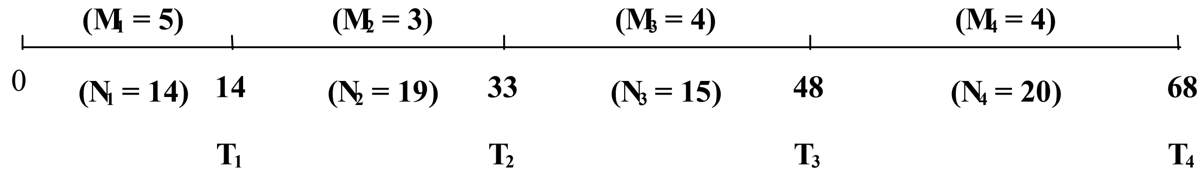

Figure 3.5 Test Data for Grouped Data Option

The solution of (54) and (55) provides mle's for λ and β corresponding to 0.595 and 0.780, respectively. Using (52) and (53) results in the following table:

TABLE C-8. ESTIMATED FAILURE RATE AND ESTIMATED RELIABILITY BY CONFIGURATION

Configuration Number, i K = 4	Estimated Failure Probability for Configuration i $\hat{f}_i$	Estimated Reliability for Configuration i $\hat{R}_i$
1	.333	.667
2	.234	.766
3	.206	.794
4	.190	.810

A plot of the estimated failure rate by configuration is:

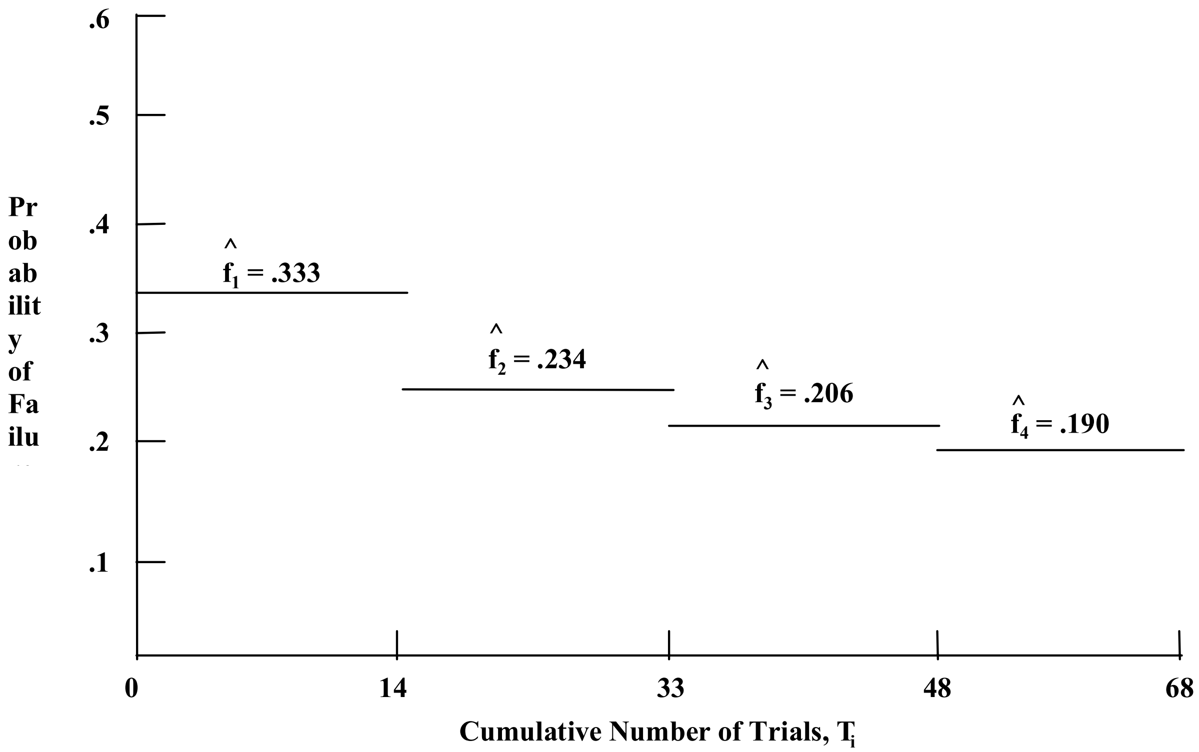

Figure 3.6 Estimated Failure Rate by Configuration

and a plot of the estimated reliability by configuration is:

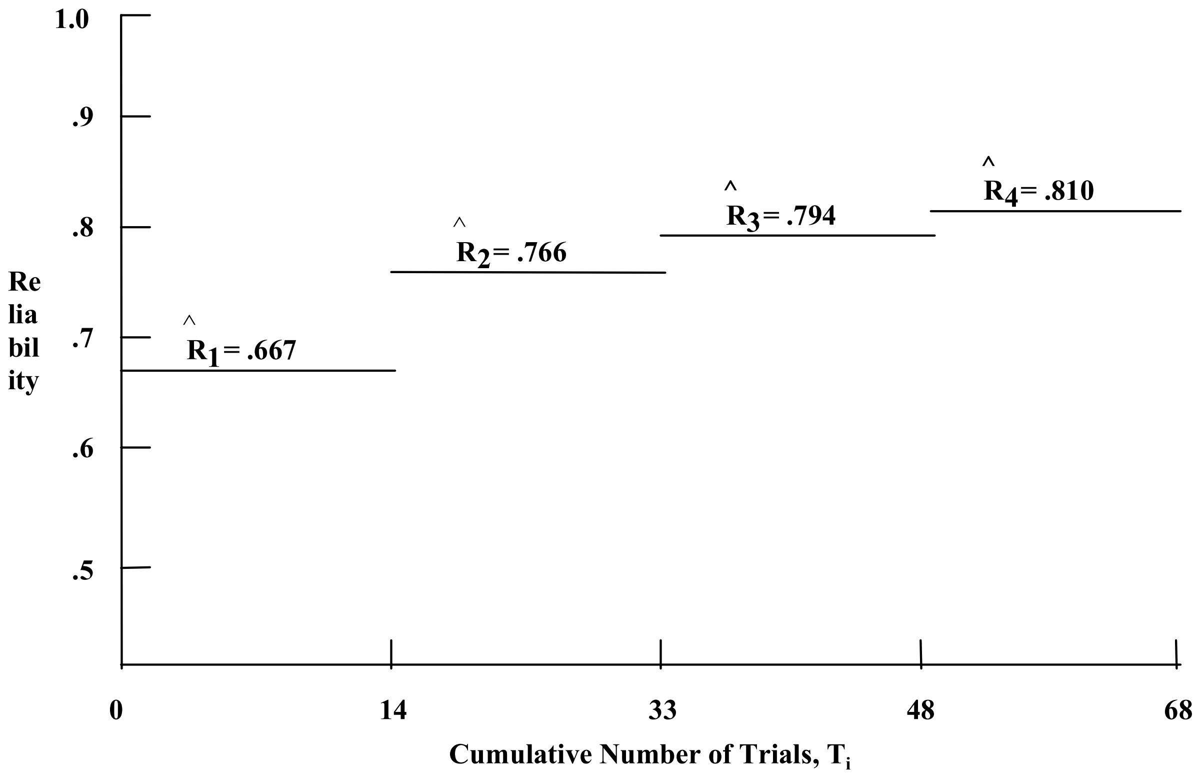

Figure 3.7 Estimated Reliability by Configuration

Finally, (56) is used to generate the following table of LCB's for the reliability of the last configuration:

TABLE C-9. TABLE OF LOWER CONFIDENCE BOUNDS (LCB'S) FOR FINAL CONFIGURATION

Confidence Level	LCB
.50	.806
.75	.783
.80	.777
.90	.761
.95	.747

3.3 Subsystem Level Reliability Growth Tracking Models.

3.3.1 AMSAA SSTRACK Model Description and Conditions For Usage. The AMSAA Subsystem Tracking Model (SSTRACK) is a tool for assessing system level reliability from lower level test results. The methodology was developed to make greater use of component or subsystem test data in estimating system reliability. By representing the system as a series of independent subsystems, the methodology permits an assessment of the system level demonstrated reliability at a given confidence level from the subsystem test data. This system level assessment is permissible provided the:

- subsystem test conditions/usage are in conformance with the proposed system level operational environment (as embodied in the Operational Mode Summary/Mission Profile [OMS/MP]) and

- Failure Definitions/Scoring Criteria (FD/SC) formulated for each subsystem are consistent with the FD/SC used for system level test evaluation.

The SSTRACK methodology supports a mix of test data from growth and non-growth subsystems. For growth subsystems, the model uses test results in the form of either individual failure times or grouped data. Statistical goodness-of-fit procedures are used for assessing model applicability for growth subsystem test data. For non-growth subsystems, the model uses fixed configuration test data in the form of the total test time and the total number of failures. The model applies the Lindström-Madden method [4] for combining the test data from the individual subsystems. Twenty-five subsystems can be represented by the current implementation of the model. SSTRACK is a continuous model, but it may be used with discrete data if the number of trials is large and the probability of failure is small.

A potential benefit of this methodology is that it may allow for reduced system level testing by combining lower level subsystem test results in such a manner that system reliability may be demonstrated with confidence. Another potential benefit is that it may allow for an assessment of the degree of subsystem test contribution toward demonstrating a system reliability requirement. Finally, as mentioned, it may serve as an effective means of combining test data from dissimilar sources, namely growth and non-growth subsystems.

Besides the two provisos stated in the opening paragraph regarding OMS/MP conformance and FD/SC consistency, a caveat in using the methodology is that high-risk subsystem interfaces should be identified and addressed through joint subsystem testing. Also, as in any reliability growth test program, growth subsystem configuration changes must be properly documented for the methodology to provide meaningful results.

The primary output from the SSTRACK computer implementation is a table of lower confidence bounds for the system reliability (MTBF) for a range of confidence levels.

3.3.2 Methodology.

LIST OF NOTATION

^	denotes an estimate when placed over a parameter
M	Mean Time Between Failures (MTBF)
D	demonstration
G	growth
LCB	Lower Confidence Bound

γ	gamma = confidence level
T	(total) test time
N	(total) number of failures
$\chi^2_{df,\gamma}$	chi-squared percentile point for df degrees of freedom and γ confidence
β	beta = growth parameter from reliability growth tracking model

To be able to handle a mix of test data from growth and non-growth subsystems, the methodology converts all growth subsystem test data to its "equivalent" amount of demonstration test time and "equivalent" number of demonstration failures so that all subsystem results are expressed in a common format; namely, in terms of fixed configuration (non-growth) test data. By treating growth subsystem test data in this way, a standard lower confidence bound formula for fixed configuration test data may be used to compute the system reliability lower confidence bound for the combination of growth and non-growth data. The net effect of this conversion process is that it reduces all growth subsystem test data to "equivalent" demonstration test data while preserving the following two important equivalency properties:

The "equivalent" demonstration data estimators and the growth data estimators must yield:

(1) the same subsystem MTBF point estimate and

(2) the same subsystem MTBF lower confidence bound.

In other words, the methodology maintains the following relationships, respectively:

$$\hat{M}_D = \hat{M}_G \tag{57}$$

$$LCB_\gamma(D) = LCB_\gamma(G) \tag{58}$$

where

$$\hat{M}_D = \frac{T_D}{N_D} \tag{59}$$

$$LCB_\gamma(D) = \frac{2T_D}{\chi^2_{2N_D+2,\gamma}} \tag{60}$$

Reducing growth subsystem test data to "equivalent" demonstration test data using the following equations closely satisfies the relationships cited above:

$$N_D = \frac{N_G}{2} \tag{61}$$

$$T_D = \hat{M}_G \times \frac{N_G}{2} = \frac{T_G}{2\hat{\beta}} \tag{62}$$

The growth estimate for the MTBF, $\hat{M}_G$, and the estimate for the growth parameter, $\hat{\beta}$, are described in the sections on point estimation for system level Continuous Reliability Growth Tracking Models.

The model then uses the above equations to compute an approximate lower confidence bound for the serial system reliability (MTBF) from non-growth subsystem demonstration data and growth subsystem "equivalent" demonstration data as described in the following section on the Lindström-Madden method.

3.3.3 Lindström-Madden Method. In addition to using the notation defined in the previous section on Methodology, subsequent equations use the following notation:

LIST OF NOTATION

sys	system level
min	minimum of
K	number of subsystems in serial system
ρ	failure rate
i	subscript for subsystem number
Σ	summation of

To compute an approximate lower confidence bound (LCB) for the system MTBF from subsystem demonstration and "equivalent" demonstration data, the AMSAA SSTRACK model uses an adaptation of the Lindström-Madden method by computing the following four estimates:

1. the equivalent amount of system level demonstration test time. (This estimate is a reflection of the least tested subsystem because it is the minimum demonstration test time of all the subsystems.),

2. the current system failure rate, which is the sum of the estimated failure rate from each subsystem i, i = 1..K,

3. the "equivalent" number of system level demonstration failures, which is the product of the previous two estimates, and

4. the approximate LCB for the system MTBF at a given confidence level, which is a function of the equivalent amount of system level demonstration test time and the equivalent number of system level demonstration failures.

In equation form, these system level estimates are, respectively:

$$T_{D,sys} \quad = \quad \min T_{D,i} \qquad for\ i \quad = \quad 1..K \tag{63}$$

$$\hat{\rho}_{sys} \quad = \quad \sum_{i=1}^{K} \hat{\rho}_i \tag{64}$$

where

$$\hat{\rho}_i \quad = \quad \frac{1}{\hat{M}_{D,i}} \tag{65}$$

$$\hat{M}_{D,i} \quad = \quad the\, current\ MTBF\ estimate\ for\ subsystem\ i$$

$$N_{D,sys} \quad = \quad \hat{\rho}_{sys} \times T_{D,sys} \tag{66}$$

$$LCB_{\gamma} \quad = \quad \frac{2T_{D,sys}}{\chi^2_{2N_{D,sys}+2,\gamma}} \tag{67}$$

3.3.4 Example. The following example is an application of the AMSAA Subsystem Level Reliability Growth Tracking Model to a system composed of three subsystems: one non-growth and two growth subsystems. Of the two growth subsystems, one has data recorded in the form of individual cumulative failure times, and the other has grouped failure data. Besides showing that SSTRACK can be used for test data gathered from dissimilar sources (namely, non-growth and growth subsystems), this particular example was chosen to show that system level reliability estimates are influenced by -

- the least tested subsystem and
- the least reliable subsystem, that is, the subsystem with the largest failure rate.

Subsystem 1 in this example is a non-growth subsystem consisting of fixed configuration data of 8,000 hours of test time and 2 observed failures.

Subsystem 2 is a growth subsystem with individual failure time data. In 300 hours of test time there were 27 observed failures occurring at the following cumulative times: 2.6, 16.5, 16.5, 17.0, 21.4, 29.1, 33.3, 56.5, 63.1, 70.6, 73.0, 77.7, 93.9, 95.5, 98.1, 101.1, 132.0, 142.2, 147.7, 149.0, 167.2, 190.7, 193.0, 198.7, 251.9, 282.5 and 286.1.

Subsystem 3 is a growth subsystem with failure data from five groups. At least 3 groups are required for goodness-of-fit purposes for this model. Although this subsystem has an equal amount of test time in each group, unequal test times are also permissible.

TABLE C-10. SUBSYSTEM 3 TEST DATA

Group Number	Start and End Time	Test Time in Group	Observed Number of Failures
1	0 20	20	13
2	20 40	20	16
3	40 60	20	5
4	60 80	20	8
5	80 100	20	7
			Total = 49

The following table shows the pertinent statistics for each subsystem i. It is here that all growth (G) subsystem test data are reduced to equivalent demonstration (D) test data.

TABLE C-11. SUBSYSTEM STATISTICS

Statistics (i = 1,2,3)	Subsystem 1 (Non-growth)	Subsystem 2 (Growth)	Subsystem 3 (Growth)
$T_{G,i}$	N/A	300	100
$N_{G,i}$	N/A	27	49
$\hat{M}_{G,i}$	N/A	15.511	2.639
$N_{D,i} = \frac{N_{G,i}}{2}$	2	13.5	24.5
$T_{D,i} = \hat{M}_{G,i} \times N_{D,i}$	8000	209.4	64.7
$\hat{M}_{D,i} = \hat{M}_{G,i} = \frac{T_{D,i}}{N_{D,i}}$	4000	15.511	2.639
$\hat{\rho}_i = \frac{1}{\hat{M}_{D,i}}$	2.50 x 10^{-4}	6.45 x 10^{-2}	3.79 x 10^{-1}

System level statistics are computed by applying the Lindström-Madden method to the equivalent demonstration data from each subsystem.

$$T_{D,sys} = \min T_{D,i(i=1,2,3)} = 64.7 \tag{68}$$

$$\hat{\rho}_{sys} = \sum_{i=1}^{3} \hat{\rho}_i = 4.44 \times 10^{-1} \tag{69}$$

$$\hat{M}_{D,sys} = \frac{1}{\hat{\rho}_{sys}} = 2.25 \tag{70}$$

$$N_{D,sys} \quad = \quad T_{D,sys} \times \hat{\rho}_{sys} \quad = \quad 28.7 \tag{71}$$

$$LCB_{.80} \quad = \quad \frac{\left(2 \times T_{D,sys}\right)}{\chi^2_{2N_{D,sys}+2,.80}} \quad = \quad 1.89 \quad \left(confidence \;\; level = 80\%\right) \tag{72}$$

Finally, a table of lower confidence bounds is shown for the system reliability (MTBF) for a range of confidence levels.

TABLE C-12. SYSTEM LOWER CONFIDENCE BOUNDS (LCB'S)

Confidence Level (in percent)	LCB for System MTBF
50	2.20
55	2.15
60	2.10
65	2.05
70	2.00
75	1.95
80	1.89
85	1.83
90	1.76
95	1.65
98	1.54
99	1.48

REFERENCES

1. MIL-HDBK-189, Reliability Growth Management, 13 February 1981

2. Crow, Larry, Methodology Office Note 1-83, AMSAA Discrete Reliability Growth Model, March 1983

3. Broemm, William, Briefing Charts, Subsystem Tracking Model (SSTRACK) (Demonstration of System Reliability from Lower Level Testing), March 1996

4. Lloyd, D. K., and M. Lipow; Reliability: Management, Methods, and Mathematics, Prentice Hall, NJ; 1962; p. 227

4. RELIABILITY GROWTH PROJECTION

4.1 Reliability Projection Concepts and Methodology. The reliability growth process applied to a complex system undergoing development involves surfacing failure modes, analyzing the modes, and implementing corrective actions (termed fixes) to the surfaced modes. In such a manner, the system configuration is matured with respect to reliability. The rate of improvement in reliability is determined by (1) the on-going rate at which new problem modes are being surfaced, (2) the effectiveness and timeliness of the fixes, and (3) the set of failure modes that are addressed by fixes.

At the end of a test phase, program management usually desires an assessment of the system's reliability associated with the current configuration. Often, the amount of data generated from testing the current system configuration is severely limited. In such circumstances, if the failure data generated over a number of system configurations is consistent with a reliability growth model, we can pool the data over the tested configurations to estimate the parameters of the growth model. This in turn will yield a reliability tracking curve that gives estimates of the configuration reliabilities. The resulting assessment of the system's current reliability is called a demonstrated estimate since it is based solely on test data.

If the current configuration is the result of applying a group of fixes to the previous configuration, there could be a statistical lack of fit in tracking reliability growth between the previous and current configurations. In such a situation it may not be valid to use a reliability growth tracking model to pool configuration data to assess the reliability of the current configuration. We always have the option of estimating the current configuration reliability based only on failure data generated for this configuration. However, such an estimate may be poor if little test time has been accumulated since the group of fixes was implemented. In this situation, program management may wish to use a reliability projection method. Such methods are typically based on assessments of the effectiveness of corrective actions and failure data generated from the current and previous configurations.

A second situation in which a reliability projection is often utilized is when a group of fixes are scheduled for implementation at the end of the current test phase, prior to commencing a follow-on test phase. Program management often desires a projection of the reliability that will be achieved by implementing the delayed fixes. This type of projection can be based solely on the current test phase failure data and engineering assessments of the effectiveness of the planned fixes. The Crow/AMSAA model in Section 4.3 or the AMSAA Maturity Projection Model (AMPM) discussed in Section 4.4 can be used to obtain such projections.

The current test phase could consist of several system configurations if not all the fixes to surfaced problem modes are delayed. In this instance we can still obtain a projection of the reliability with which the system will enter the follow-on test by using the AMPM.

Another situation in which a projection can be useful is in assessing the plausibility of meeting future reliability milestones, i.e., milestones beyond the commencement of the follow-on test. The AMPM can provide such projections based on failure data generated to date and fix effectiveness assessments for all implemented and planned fixes to surfaced problem modes.

In Section 4.2 we present several basic concepts used in connection with our reliability projection models. We also establish notation and present assumptions that are used throughout this section. Notation and assumptions directed toward a particular method are introduced in the corresponding section.

In Sections 4.3 and 4.4 we present two reliability projection models and associated statistical procedures. In Section 4.3 we discuss the Crow/AMSAA model. This model is used to estimate the system failure intensity at the beginning of a follow-on test phase based on information from the previous test phase. This information consists of problem mode first occurrence times, the number of failures associated with each problem mode, and the total number of failures due to modes that will not be addressed by fixes. Additionally, the projection uses engineering assessments of the planned corrective actions to problem modes surfaced during the test phase. The associated statistical estimation procedure assumes that all the corrective actions are implemented at the end of the current test phase but prior to commencing the follow-on test phase. This model addresses the continuous case, i.e., where test duration is measured in a continuous fashion such as in hours or miles.

In Section 4.4 we present another reliability projection model that addresses the continuous case. This model is called the AMSAA Maturity Projection Model – Continuous (AMPM-Continuous). The model can be applied to the situation where one wishes to utilize test data generated over one or more test phases to project the impact of fixes to surfaced problem failure modes. The model does not require that the fixes be all delayed to the end of the current test phase. It only assumes the fixes are implemented prior to the time at which a projection is desired. Also, projections may be made for milestones beyond the start of the next test phase. The section contains an example application of the AMPM.

4.2 Basic Concepts, Notation and Assumptions. Throughout this section we shall regard a potential failure mode as consisting of one or more potential failure sites with associated failure mechanisms. Fixes are often applied to failure modes surfaced through testing. As in [Reference 1], we shall define a B-mode to be a failure mode we would apply a fix to if the mode were surfaced. All other failure modes will be referred to as A-modes. A surfaced mode might be regarded as an A-mode if (1) a fix is not economically justifiable, or (2) the underlying failure mechanisms associated with the mode are not sufficiently understood to attempt a fix. Thus the rate of failure due to the set of A-modes is constant as long as the failure modes are not reclassified.

For a surfaced B-mode, the rate of occurrence would hopefully diminish after implementing a fix to the mode. However, in general, we cannot expect the mode rate of occurrence to drop to zero. Fixes are seldom perfect; for example, our fix may not eliminate all the potential failure mechanisms associated with the B-mode. Thus, for each B-mode, say mode i, we associate a fix effectiveness factor (FEF), denoted by d_i. The FEF d_i is the fraction by which the initial rate of occurrence of mode i is reduced due to the fix. The assessed values for the d_i of surfaced B-modes are often based largely on engineering judgement. This is why the corresponding reliability assessment is termed a "projection" as opposed to a "demonstrated value" that is based solely on the test data.

List of Notation:

K	Number of potential B-modes that reside in the system
λ_i	Initial rate of occurrence of B-mode I $(i=1,\cdots,K)$
λ_A	Contribution of A-modes to system failure intensity
λ_B	B-mode contribution to initial system failure intensity
T	Total duration of conducted test. Typically measured in hours or miles.
N_A	Number of A-mode failures that occur over [0,T]
N_B	Number of B-mode failures that occur over [0,T]
m	Number of distinct B-modes surfaced over [0,T]
$M(t)$	Random variable of number of distinct B-modes surfaced by test duration t
$\mu(t)$	The expected value of $M(t)$
t_i	Time of first occurrence of B-mode i $(i=1,\cdots,K)$
$\underline{t}$	Vector of B-mode first occurrence times $(t_1,\cdots,t_m)$
N_i	Number of failures associated with B-mode i that occurs during test
d_i	Fix effectiveness factor (FEF) for B-mode i. The factor d_i is the fraction of λ_i removed by the fix.
μ_d	Arithmetic average of the d_i, i.e., $\left(1/K\right)\sum_{i=1}^{K} d_i$
obs	The index set associated with the m B-modes that are surfaced during test

E	Expectation operator
V	Variance operator
mle	Maximum likelihood estimator
^	When placed over a parameter, it denotes an estimate
~	"Distributed as"
≈	"Approximated by"
≅	"Approximately equal to"

Assumptions:

1. At the start of test, there is a large unknown constant number, denoted by K, of potential B-modes that reside in the system (which could be a complex subsystem).

2. Failure modes (both types A and B) occur independently.

3. Each occurrence of a failure mode results in a system failure.

4. No new modes are introduced by attempted fixes.

Additional notation and assumptions germane to a particular model will be introduced in the section dealing with the model.

4.3 Crow/AMSAA Reliability Projection Model.

4.3.1 Introduction. In this section we shall consider the case where all fixes to surfaced B-modes are implemented at the end of the current test phase prior to commencing a follow-on test phase. Thus all fixes are delayed fixes. The current test phase will be referred to as Phase I and the follow-on test phase as Phase II.

The Crow/AMSAA reliability projection model and associated parameter estimation procedure was developed to assess the reliability impact of a group of delayed fixes. In particular, the model and estimation procedure allow assessment of what the system failure intensity will be at the start of Phase II after implementation of the delayed fixes. Denoting this failure intensity by r(T), where T denotes the duration of Test Phase I, the Crow/AMSAA assessment of r(T) is based on: (1) the A and B mode failure data generated during Phase I test duration T; and (2) assessments of the fix effectiveness factors (FEFs) for the B-modes surfaced during Phase I. Since the assessments of the FEFs are often largely based on engineering judgement, the resulting assessment, $\hat{r}(T)$, of the system failure intensity after fix implementations is called a reliability projection as opposed to a demonstrated assessment (which would be based solely on test data).

The Crow/AMSAA projection model and estimation procedure was motivated by the desire to replace the widely used "adjustment procedure." The adjustment procedure assessed r(T) based on reducing the number of failures N_i due to B-mode i during Phase

I to $\left(1-d_i^*\right)N_i$, where d_i^* is the assessment of d_i. Note $\left(1-d_i^*\right)N_i$ is an assessment of the expected number of failures due to B-mode i that would occur in a follow-on test of the same duration as Phase I. The adjustment procedure assesses r(T) by $\hat{r}_{adj}(T)$ where

$$\hat{r}_{adj}(T) \quad = \quad \frac{N_A}{T} + \frac{\sum_{i \in obs}\left(1-d_i^*\right)N_i}{T} \tag{1}$$

Crow[1] shows that even if the assessed FEFs are equal to the actual d_i, the adjustment procedure systematically underestimates r(T). This bias, i.e.,

$$B(T) \quad = \quad E\{r(T)-\hat{r}_{adj}(T)\} \quad > \quad 0 \tag{2}$$

is calculated in [1] by considering the random set of B-modes surfaced during Phase I. In particular, the adjustment procedure is shown to be biased since it fails to take into account that, in general, not all the B-modes will be surfaced by the end of Phase I. Before discussing how the Crow/AMSAA methodology addresses this bias we shall list some additional notation and assumptions associated with the Crow/AMSAA model.

4.3.2 Crow/AMSAA Model Notation and Additional Assumptions.

List of Notation:

T	Length of Test Phase I
r(T)	System failure intensity at beginning of Test Phase II after implementation of delayed B-mode fixes. Viewed as a random variable whose value is determined by the set of B-modes surfaced during Test Phase I and the associated fix effectiveness factors.
$\rho(T)$	Expected value of r(T) with respect to random set of B-modes surfaced in Test Phase I, conditioned on the fix effectiveness factor values. We write $\rho(T)=E(r(T))$.
$\hat{r}_{adj}(T)$	Adjustment procedure assessment of the value taken on by r(T)
B(T)	Bias incurred by assessing the value of r(T) by $\hat{r}_{adj}(T)$ Thus, $B(T)=E\{r(T)-\hat{r}_{adj}(T)\}$
ρ_{GP}	Growth potential system failure intensity
M_{GP}	Growth potential system MTBF, i.e., $M_{GP}=\left(\rho_{GP}\right)^{-1}$

$h(t)$ Expected rate of occurrence of new B-modes at test duration t. Note:

$$h(t) = \frac{d\,\mu(t)}{d\,t}$$

$h_c(t), r_c(t), \rho_c(t)$ Crow/AMSAA model approximations to $h(t)$, r(t), $\rho(t)$ respectively

$M(T), M_c(T)$ Denote $(\rho(T))^{-1}$ and $(\rho_c(T))^{-1}$ respectively

Additional Assumptions for Crow/AMSAA:

1. The time to first occurrence is exponentially distributed for each failure mode.

2. No fixes to B-modes are implemented during Test Phase I. Fixes to all B-modes surfaced during Phase I are implemented prior to Phase II.

3. The fix effectiveness factors (FEFs) $d_i,\ i \in obs$, associated with the B-modes surfaced during Phase I are realized values of a set of random variables $\{D_i \mid i \in obs\}$ where:
   (a) The D_i are independent;
   (b) The D_i have common mean value μ_d; and
   (c) The D_i are independent of $M(T)$.

4. The random process for the number of distinct B-modes that occur over test interval $[0, t]$, i.e. $M(t)$, is well approximated by a non-homogeneous Poisson process with mean value function $\mu_c(t) = \lambda t^{\beta}$ for some $\lambda, \beta > 0$.

4.3.3 Crow/AMSAA Model Equations and Estimation Procedure. The Crow/AMSAA model assesses the value of the system failure intensity, r(T), after implementation of the Phase I delayed fixes. This assessment is taken to be an estimate of the expected value of r(T), i.e., an estimate of $\rho(T) = E(r(T))$. In [1] (and in section 4.4.3) it is shown that:

$$\rho(T) \quad = \quad \lambda_A + \sum_{i=1}^{K} (1 - d_i)\lambda_i + \sum_{i=1}^{K} d_i\, \lambda_i e^{-\lambda_i T} \qquad (3)$$

The traditional adjustment procedure assessment for the value of r(T) is actually an estimate of

$$\lambda_A + \sum_{i=1}^{K} (1 - d_i)\, \lambda_i$$

since as shown later in this subsection

$$E\left(\hat{r}_{adj}(T)\right) \;=\; \lambda_A + \sum_{i=1}^{K}\left(1-d_i^*\right)\lambda_i \qquad (4)$$

where d_i^* is an assessment of d_i. Thus, by (3) and (4), the adjustment procedure has the bias B(T) where

$$\begin{aligned} B(T) &= E\{r(T) - \hat{r}_{adj}(T)\} \\ &= \rho(T) - E\left(\hat{r}_{adj}(T)\right) \\ &= \sum_{i=1}^{K}\left(d_i^* - d_i\right)\lambda_i + \sum_{i=1}^{K} d_i\,\lambda_i\,e^{-\lambda_i T} \end{aligned}$$

It follows that for $d_i^* = d_i \;\; (i = 1, \cdots, K)$

$$B(T) \;=\; \sum_{i=1}^{K} d_i\,\lambda_i\,e^{-\lambda_i T} \qquad (5)$$

This shows that even with perfect knowledge of the d_i (i.e., when $d_i^* = d_i$), the adjustment procedure provides a biased underestimate of the value of r(T). The Crow/AMSAA procedure attempts to reduce this bias by estimating B(T) given by (5).

To estimate B(T), the Crow/AMSAA Model uses an approximation to B(T). This approximation is obtained in two steps. The first step is to regard the d_i in (5) as realizations of random variables $D_i \;\; (i = 1, \cdots, K)$ that satisfy assumption number 3 in the "Additional Assumptions for Crow/AMSAA." Then B(T) is approximated by the expected value (with respect to the D_i) of

$$\sum_{i=1}^{K} D_i\,\lambda_i\,e^{-\lambda_i T}$$

Thus the initial approximation arrived at for B(T) in (5) is

$$\begin{aligned} B(T) &\approx E\left(\sum_{i=1}^{K} D_i\,\lambda_i\,e^{-\lambda_i T}\right) \\ &= \mu_d \sum_{i=1}^{K} \lambda_i\,e^{-\lambda_i T} \qquad (6) \end{aligned}$$

where $\mu_d = E(D_i)$ $(i = 1, \cdots, K)$. The final step to obtain the Crow/AMSAA approximation of B(T) is to replace the sum

$$\sum_{i=1}^{K} \lambda_i \, e^{-\lambda_i T}$$

in (6) by a two parameter function of T. The Crow/AMSAA Model replaces this sum by the power function

$$h_c(T) \; = \; \lambda \, \beta \, T^{\beta - 1} \qquad\qquad for \quad \lambda, \beta > 0 \tag{7}$$

The form in (7) is chosen based on the desire for a mathematically tractable estimation problem and an empirical observation. Based on an empirical study, Crow [1] states that the number of distinct B-modes surfaced over a test period $[0, t]$ can often be approximated by a power function of the form

$$\mu_c(t) \; = \; \lambda \, t^{\beta} \qquad\qquad for \quad \lambda, \beta > 0 \tag{8}$$

In (8), Crow [1] interprets $\mu_c(t)$ as the expected number of distinct B-modes surfaced during the test interval $[0, t]$. More specifically, [1] assumes the number of distinct B-modes occurring over $[0, t]$ is governed by a non-homogeneous Poisson process with $\mu_c(t)$ as the mean value function. Thus

$$h_c(t) \; = \; \frac{d\, \mu_c(t)}{d\, t} \; = \; \lambda \, \beta \, t^{\beta - 1} \tag{9}$$

represents the expected rate at which new B-modes are occurring at test time t.

In Annex 1 of Appendix D, under the previously stated assumptions, it is shown that the expected number of distinct B-modes surfaced over $[0, t]$ is given by

$$\mu(t) \; = \; \sum_{i=1}^{K} \left(1 - e^{-\lambda_i t}\right) \tag{10}$$

Thus the expected rate of occurrence of new B-modes at test time t is

$$h(t) \; = \; \frac{d\, \mu(t)}{d\, t} \; = \; \sum_{i=1}^{K} \lambda_i e^{-\lambda_i t} \tag{11}$$

Equation (11) shows that the initial approximation to the bias B(T), given in (6) can be expressed as

$$B(T) \approx \mu_d \, h(T) \tag{12}$$

By replacing h(T) in (12) by $h_c(T)$ given in (9), we arrive at the final Crow/AMSAA Model approximation to B(T), namely

$$\begin{aligned} B_c(T) &= \mu_d \, h_c(T) \\ &= \mu_d \, \lambda \, \beta T^{\beta-1} \end{aligned} \tag{13}$$

Returning to our expression in (3) for the expected value of the system failure intensity after incorporation of the Phase I delayed fixes, i.e., $\rho(T) = E(r(T))$., we can now write down the Crow/AMSAA Model approximation for $\rho(T)$. This approximation, by (13), is given by:

$$\begin{aligned} \rho_c(T) &= \lambda_A + \sum_{i=1}^{K} (1-d_i)\lambda_i + B_c(T) \\ &= \lambda_A + \sum_{i=1}^{K} (1-d_i)\lambda_i + \mu_d \left(\lambda \, \beta T^{\beta-1}\right) \end{aligned} \tag{14}$$

We shall next consider the Crow/AMSAA procedure for estimating $\rho_c(T)$. This estimate is taken as the assessment of the system failure intensity after incorporation of the delayed fixes.

Consider the first term in the expression for $\rho_c(T)$ given in (14), i.e., λ_A. Since the A-modes are not fixed, the A-mode failure rate λ_A is constant over [0,T]. Thus we simply estimate λ_A by

$$\hat{\lambda}_A = \frac{N_A}{T} \tag{15}$$

where N_A is the number of A-mode failures over [0,T]. Note

$$E(\hat{\lambda}_A) = \frac{E(N_A)}{T} = \frac{\lambda_A T}{T} = \lambda_A \tag{16}$$

Next consider estimation of the summation

$$\sum_{i=1}^{K}(1-d_i)\lambda_i$$

in the expression for $\rho_c(T)$. By the second assumption in the "Additional Assumptions for Crow/AMSAA," all fixes are delayed until Test Phase I has been completed. This implies the failure rate for B-mode i $(i=1,\cdots,K)$ remains constant over [0,T]. Thus we simply estimate λ_i by

$$\hat{\lambda}_i = \frac{N_i}{T} \qquad (i=1,\cdots,K) \tag{17}$$

where N_i denotes the number of failures during [0,T] attributable to B-mode i. Note

$$E(\hat{\lambda}_i) = \frac{E(N_i)}{T} = \frac{\lambda_i T}{T} = \lambda_i \tag{18}$$

Equations (16) and (18) suggest we assess

$$\lambda_A + \sum_{i=1}^{K}(1-d_i)\lambda_i$$

by

$$\begin{aligned} \hat{r}_{adj}(T) &= \hat{\lambda}_A + \sum_{i=1}^{K}(1-d_i^*)\hat{\lambda}_i \\ &= \frac{N_A}{T} + \sum_{i=1}^{K}(1-d_i^*)\left(\frac{N_i}{T}\right) \end{aligned} \tag{19}$$

Observe $N_i = 0$ if B-mode i does not occur during [0,T]. Thus

$$\hat{r}_{adj}(T) = \frac{N_A}{T} + \sum_{i\in obs}(1-d_i^*)\left(\frac{N_i}{T}\right) \tag{20}$$

where obs = {i | B-mode i occurs during [0,T]}. Note the adjustment procedure estimate has the form

$$\hat{r}_{adj}(T) = \frac{N^*}{T} \tag{21}$$

where

$$N^* = N_A + \sum_{i \in obs} \left(1 - d_i^*\right) N_i$$

(22)

is the "adjusted" number of failures.

For given fix effectiveness factor (FEF) assessments, d_i^*, note that

$$E\left(\hat{r}_{adj}(T)\right) = T^{-1}\left\{E(N_A) + \sum_{i=1}^{K}\left(1 - d_i^*\right)E(N_i)\right\}$$

$$= T^{-1}\left\{\lambda_A T + \sum_{i=1}^{K}\left(1 - d_i^*\right)(\lambda_i T)\right\}$$

$$= \lambda_A + \sum_{i=1}^{K}\left(1 - d_i^*\right)\lambda_i \qquad (23)$$

Thus, as stated earlier, we see that the adjustment procedure estimate only provides an assessment for a portion of the expected system failure intensity, namely

$$\lambda_A + \sum_{i=1}^{K}\left(1 - d_i\right)\lambda_i$$

Returning to the fundamental equation for the Crow/AMSAA Model approximation to the expected system failure intensity, i.e. (14),

$$\rho_c(T) = \lambda_A + \sum_{i=1}^{K}\left(1 - d_i\right)\lambda_i + \mu_d\left(\lambda \beta T^{\beta-1}\right)$$

Let us next consider the assessment of the fix effectiveness factors d_i. The assessment d_i^* will often be based largely on engineering judgement. The value chosen for d_i^* should reflect several considerations:
(1) How certain we are that the root cause for B-mode i has been correctly identified; (2) the nature of the fix, e.g., its complexity; (3) past FEF experience; and (4) any germane testing (including assembly level testing).

Note that (20) shows that we need only assess FEFs for those B-modes that occur during [0,T] to make an assessment of

$$\lambda_A + \sum_{i=1}^{K}\left(1 - d_i\right)\lambda_i$$

To assess the mean FEF, $\mu_d = E(D_i)$, we utilize our assessments d_i^* for $i \in obs$. Let m be the number of distinct B-modes surfaced over [0,T]. Then we assess μ_d by

$$\mu_d^* = \frac{1}{m} \sum_{i \in obs} d_i^* \tag{24}$$

To complete our assessment of the expected system failure intensity after incorporation of delayed fixes, we shall now address the assessment of

$$h_c(T) = \lambda \beta T^{\beta - 1}$$

To develop a statistical estimation procedure for λ and β, the Crow/AMSAA Model regards the number of distinct B-modes occurring in an interval [0,t], denoted by $M(t)$, as a random process. The model assumes that this random process can be well approximated, for large K, by a non-homogeneous Poisson process with mean value function

$$\mu_c(t) = E(M(t)) = \lambda t^{\beta}$$

where λ, β, $t > 0$. As noted earlier in (9)

$$h_c(t) = \frac{d\,\mu_c(t)}{d\,t}$$

The data required to estimate λ and β are (1) the number of distinct B-modes, m, that occur during [0,T] and (2) the B-mode first occurrence times $0 < t_1 \le t_2 \le \cdots \le t_m \le T$. Crow [1] states that the maximum likelihood estimates of λ and β, denoted by $\hat{\lambda}$ and $\hat{\beta}$ respectively, satisfy the following equations:

$$\hat{\lambda} T^{\hat{\beta}} = m \tag{25}$$

$$\hat{\beta} = \frac{m}{\sum_{i=1}^{m} \ln\left(\frac{T}{t_i}\right)} \tag{26}$$

Note (25) merely says that the estimated number of distinct B-modes that occur during [0,T] should equal the observed number of distinct B-modes over this period. Solving (25) for $\hat{\lambda}$ we can write our estimate for $h_c(T)$ in terms of m and $\hat{\beta}$ as follows:

$$\hat{h}_c(T) \quad = \quad \hat{\lambda}\,\hat{\beta}\,T^{\hat{\beta}-1} \quad = \quad \left(\frac{m}{T^{\hat{\beta}}}\right)\hat{\beta}\,T^{\hat{\beta}-1}$$

$$= \quad \frac{m\,\hat{\beta}}{T} \tag{27}$$

Crow [1] notes that conditioned on the observed number of distinct B-modes m ,i.e. $M(T) \quad = \quad m$, the estimator

$$\overline{\beta}_m \quad = \quad \left(\frac{m-1}{m}\right)\hat{\beta} \qquad\qquad m \geq 2 \tag{28}$$

is an unbiased estimator of β, i.e.,

$$E\left(\overline{\beta}_m\right) \quad = \quad \beta \tag{29}$$

Thus we shall also consider estimating $h_c(T) \quad = \quad \lambda\,\beta\,T^{\beta-1}$ by using $\overline{\beta}_m$. This leads to the estimate

$$\overline{h}_c(T) \quad = \quad \frac{m\,\overline{\beta}_m}{T} \tag{30}$$

Finally, to complete our assessment of the system failure intensity, we need to assess the Crow/AMSAA Model expected system failure intensity $\rho_c(T)$. Recall, by (14)

$$\rho_c(T) \quad = \quad \lambda_A + \sum_{i=1}^{K}(1-d_i)\,\lambda_i + \mu_d\,h_c(T) \tag{31}$$

Piecing together our assessments for the individual terms in (31) we arrive at the following assessment for $\rho_c(T)$ based on $\hat{\beta}$:

$$\hat{\rho}_c(T) \quad = \quad \frac{N_A}{T} + \sum_{i=1}^{K}\left(1-d_i^*\right)\left(\frac{N_i}{T}\right) + \left(\frac{1}{m}\sum_{i\in obs} d_i^*\right)\left(\frac{m\,\hat{\beta}}{T}\right)$$

$$= \quad \frac{1}{T}\left\{N_A + \sum_{i=1}^{K}\left(1-d_i^*\right)N_i + \hat{\beta}\sum_{i\in obs} d_i^*\right\}$$

Since $N_i = 0$ for $i \notin obs$, we finally obtain

$$\hat{\rho}_c(T) = \frac{1}{T}\left\{N_A + \sum_{i \in obs}(1-d_i^*)\,N_i + \hat{\beta}\sum_{i \in obs} d_i^*\right\} \tag{32}$$

Likewise, we arrive at the following alternate assessment for $\rho_c(T)$ based on $\overline{\beta}_m$ (provided $m \geq 2$):

$$\overline{\rho}_c(T) = \frac{1}{T}\left\{N_A + \sum_{i \in obs}(1-d_i^*)N_i + \overline{\beta}_m \sum_{i \in obs} d_i^*\right\} \tag{33}$$

Note both estimates of $\rho_c(T)$ are of the form

$$Estimate\ \rho_c(T) = \frac{1}{T}\left\{N^* + (estimate\ \beta)\sum_{i \in obs} d_i^*\right\} \tag{34}$$

where N^* is the "adjusted" number of failures over [0,T]. Recall the historically used adjustment procedure assessment for the system failure intensity, after incorporation of delayed fixes, is given by

$$\hat{r}_{adj}(T) = \frac{N^*}{T}$$

Also recall

$$\overline{\beta}_m = \left(\frac{m-1}{m}\right)\hat{\beta} < \hat{\beta}$$

Thus we see by (32) and (33)

$$\hat{r}_{adj}(T) < \overline{\rho}_c(T) < \hat{\rho}_c(T) \tag{35}$$

Also of interest is an assessment of the reciprocal of $\rho_c(T)$, i.e.

$$M_c(T) = \{\rho_c(T)\}^{-1}$$

The assessment for the system mean time between failures after incorporation of the delayed fixes, denoted by M(T), is taken to be the Crow/AMSAA Model assessment of $M_c(T)$. The assessments of $M_c(T)$ based on $\hat{\rho}_c(T)$ and $\overline{\rho}_c(T)$ are denoted by $\hat{M}_c(T)$ and $\overline{M}_c(T)$ respectively. Thus

$$\hat{M}_c(T) = \{\hat{\rho}_c(T)\}^{-1} \tag{36}$$

and

$$\overline{M}_c(T) \;=\; \{\overline{\rho}_c(T)\}^{-1} \tag{37}$$

By (35) we have

$$\hat{M}_c(T) \;<\; \overline{M}_c(T) \;<\; \{\hat{r}_{adj}(T)\}^{-1} \tag{38}$$

In Section 4.3.5 we shall argue that $\overline{\rho}_c(T)$ generally provides a more accurate assessment of $\rho_c(T)$ than does $\hat{\rho}_c(T)$. However, somewhat surprisingly at first thought, in Section 4.3.5 we identify conditions under which $\hat{M}_c(T)$ generally provides a more accurate assessment of $M_c(T)$ than does $\overline{M}_c(T)$.

4.3.4 Reliability Growth Potential. Consider the expression in (3) for $\rho(T)$, the expected system failure intensity after incorporation of the delayed fixes. If we let $T \to \infty$ and denote the resulting limit of $\rho(T)$ by ρ_{GP} we obtain

$$\rho_{GP} \;=\; \lim_{T\to\infty} \rho(T) \;=\; \lambda_A + \sum_{i=1}^{K} (1 - d_i)\lambda_i \tag{39}$$

The expression ρ_{GP} is called the growth potential failure intensity. Its reciprocal is referred to as the growth potential MTBF. The growth potential MTBF represents a theoretical upper limit on the system MTBF. This limit corresponds to the MTBF that would result if all B-modes were surfaced and corrected with specified fix effectiveness factors. Note ρ_{GP} is estimated by

$$\hat{\rho}_{GP} \;=\; \frac{1}{T}\left(N_A + \sum_{i \in obs} (1 - d_i^*) N_i \right) \tag{40}$$

If the reciprocal $(\hat{\rho}_{GP})^{-1}$ lies below the goal MTBF then this may indicate that achieving the goal is high risk.

4.3.5 Use of the Maximum Likelihood Estimator versus the Unbiased Estimator for β. Recall that the estimator

$$\overline{\beta}_m \;=\; \left(\frac{m-1}{m}\right)\hat{\beta}$$

conditioned on $M(T) = m$, with $m \geq 2$, is unbiased for β, i.e.

$$E(\bar{\beta}_m) = \beta$$

Furthermore the variances of $\bar{\beta}_m$ and $\hat{\beta}$, denoted by $V(\bar{\beta}_m)$ and $V(\hat{\beta})$ respectively, satisfy the following:

$$V(\bar{\beta}_m) = V\left(\left(\frac{m-1}{m}\right)\hat{\beta}\right)$$

$$= \left(\frac{m-1}{m}\right)^2 V(\hat{\beta}) < V(\hat{\beta}) \qquad (41)$$

for $m \geq 2$. Equation (41) together with the unbiased property of $\bar{\beta}_m$, suggest that $\bar{\beta}_m$ provides a more accurate assessment of β than does $\hat{\beta}$.

Next consider the assessments of $h_c(T)$ based on $\hat{\beta}$ and $\bar{\beta}_m$. Recall the Crow/AMSAA Model assumes that M(t), t>0, is a non-homogeneous Poisson process with mean value function

$$\mu(t) = E(M(t)) = \lambda t^{\beta} \qquad \lambda, \beta > 0$$

Thus, in particular, M(T) is Poisson distributed with mean

$$E(M(T)) = \lambda T^{\beta}$$

Using this fact, it can be shown that $\bar{h}_c(T)$ is an approximately unbiased estimator of β under most conditions of practical interest, where it is understood that $\bar{h}_c(T)$ denotes a conditional estimator, conditioned on $M(T) \geq 2$. To be more explicit, $\bar{h}_c(T)$, when viewed as an estimator (as opposed to an estimated value), is a random variable which is a function of M(T) and the random vector of B-mode first occurrence times $(T_1, \cdots, T_{M(T)})$. When $M(T) = m$ and $(T_1, \cdots, T_{M(T)}) = (t_1, \cdots, t_m)$, the estimator $\bar{h}_c(T)$ takes on the value $\dfrac{m\,\bar{\beta}_m}{T}$

where

$$\bar{\beta}_m = \left(\frac{m-1}{m}\right)\hat{\beta} = \left(\frac{m-1}{m}\right)\left(\frac{m}{\sum_{i=1}^{m} \ln\left(\frac{T}{t_i}\right)}\right) = \frac{m-1}{\sum_{i=1}^{m} \ln\left(\frac{T}{t_i}\right)}$$

The estimator $\bar{h}_c(T)$ can be shown to satisfy the following:

$$E(\bar{h}_c(T)) \cong h_c(T) \tag{42}$$

provided $\Pr(M(T)=0) \cong 0$, where Pr denotes the probability function for M(T).

Next, consider the variances of the estimators $\bar{h}_c(T)$ and $\hat{h}_c(T)$ conditioned on $M(T)=m$. For $m \geq 2$,

$$V(\bar{h}_c(T)|\, M(T)=m) = V\left(\frac{m\,\bar{\beta}_m}{T}\right)$$

$$= \left(\frac{m}{T}\right)^2 V(\bar{\beta}_m) = \left(\frac{m}{T}\right)^2 V\left(\left(\frac{m-1}{m}\right)\hat{\beta}\right)$$

$$= \left(\frac{m}{T}\right)^2 \left(\frac{m-1}{m}\right)^2 V(\hat{\beta}) = \left(\frac{m-1}{T}\right)^2 V(\hat{\beta})$$

$$< \left(\frac{m}{T}\right)^2 V(\hat{\beta}) = V\left(\frac{m\hat{\beta}}{T}\right)$$

$$= V(\hat{h}_c(T)|\, M(T)=m) \tag{43}$$

Now consider the variances of $\bar{h}_c(T)$ and $\hat{h}_c(T)$ conditioned on $M(T)\geq 2$. Since (43) holds for each $m \geq 2$, we have

$$V(\bar{h}_c(T)|\, M(T)\geq 2) < V(\hat{h}_c(T)|\, M(T)\geq 2)$$

(44)

Equations (42) and (44) suggest that the estimator $\bar{h}_c(T)$ provides a more accurate estimate of $h_c(T)$ than does the estimator $\hat{h}_c(T)$ when two or more distinct B-modes occur during [0,T].

Next we consider the bias of the estimators $\hat{\rho}_c(T)$ and $\bar{\rho}_c(T)$. To do so, let

$$\tilde{\rho}_c(T) = \frac{1}{T}\left\{N_A + \sum_{i\in obs}(1-d_i^*)N_i + \tilde{\beta}\sum_{i\in obs} d_i^*\right\}$$

where $\tilde{\beta} \in \{\hat{\beta}, \bar{\beta}_m\}$. Also let $\tilde{h}_c(T) = \frac{m\tilde{\beta}}{T}$. By (27) and (30) we have

$$\tilde{\rho}_c(T) = \left\{\frac{N_A}{T} + \sum_{i \in obs}(1 - d_i^*)\frac{N_i}{T} + \left(\frac{1}{m}\sum_{i \in obs} d_i^*\right)\left(\frac{m\tilde{\beta}}{T}\right)\right\}$$

$$= \frac{N_A}{T} + \sum_{i \in obs}(1 - d_i^*)\frac{N_i}{T} + \left(\frac{1}{m}\sum_{i \in obs} d_i^*\right)\tilde{h}_c(T)$$

Thus the expected value of $\tilde{\rho}_c(T)$ is

$$E(\tilde{\rho}_c(T)) = \lambda_A + \sum_{i \in obs}(1 - d_i^*)\lambda_i + \left(\frac{1}{m}\sum_{i \in obs} d_i^*\right)E(\tilde{h}_c(T)) \tag{45}$$

Recall by (31),

$$\rho_c(T) = \lambda_A + \sum_{i=1}^{K}(1 - d_i)\lambda_i + \mu_d\, h_c(T)$$

Thus by equation (45), we have

$$E(\tilde{\rho}_c(T)) - \rho_c(T) =$$
$$\left\{\sum_{i \in obs}(1 - d_i^*)\lambda_i - \sum_{i=1}^{K}(1 - d_i)\lambda_i\right\} + \left(\frac{1}{m}\sum_{i \in obs} d_i^* - \mu_d\right)E(\tilde{h}_c(T)) + \mu_d\,\{E(\tilde{h}_c(T)) - h_c(T)\}$$

This shows that the portion of the bias of the estimator $\tilde{\rho}_c(T)$ that is not influenced by the assessments d_i^* is simply

$$E(\tilde{h}_c(T)) - h_c(T)$$

To reduce the bias as much as possible for a given set of fix effectiveness assessments d_i^*, we wish to make the bias

$$E(\tilde{h}_c(T)) - h_c(T)$$

as small as possible. Since $\bar{h}_c(T)$ is almost an unbiased estimator for $h_c(T)$, this suggests we use

$$\overline{\rho}_c(T) \;=\; \frac{1}{T}\left\{N_A + \sum_{i\in obs}\left(1-d_i^*\right)N_i + \overline{\beta}_m \sum_{i\in obs} d_i^*\right\}$$

to assess $\rho_c(T)$.

Next , we discuss the assessment of $M_c(T) \;=\; \{\rho_c(T)\}^{-1}$. To do so let $\widetilde{M}_c(T) \;=\; \{\widetilde{\rho}_c(T)\}^{-1}$. Thus

$$\widetilde{M}_c(T) \;=\; \left[\frac{1}{T}\left\{N_A + \sum_{i\in obs}\left(1-d_i^*\right)N_i + \widetilde{\beta}\sum_{i\in obs} d_i^*\right\}\right]^{-1}$$

$$= \left[\frac{N_A}{T} + \sum_{i\in obs}\left(1-d_i^*\right)\frac{N_i}{T} + \left(\frac{1}{m}\sum_{i\in obs} d_i^*\right)\widetilde{h}_c(T)\right]^{-1} \qquad (46)$$

Note

$$M_c(T) \;=\; \{\rho_c(T)\}^{-1}$$

$$= \left[\lambda_A + \sum_{i=1}^{K}\left(1-d_i\right)\lambda_i + \mu_d h_c(T)\right]^{-1}$$

Suppose $\lambda_A \cong 0$ and $d_i \cong 1$ for $i = 1,\cdots,K$. Then

$$M_c(T) \;\cong\; \{h_c(T)\}^{-1} \qquad (47)$$

For such conditions one may have $\dfrac{N_A}{T} \cong 0$ and $d_i^* \cong 1$ for $i \in obs$. If this is the case then

$$\widetilde{M}_c(T) \;\cong\; \{\widetilde{h}_c(T)\}^{-1} \qquad (48)$$

It can be shown that if the expected number of distinct B-modes is at least two then

$$\{h_c(T)\}^{-1} \;<\; \{\hat{h}_c(T)\}^{-1} \;<\; \{\overline{h}_c(T)\}^{-1} \qquad (49)$$

Thus, under conditions for which (47) and (48) hold, Equation (49) suggests using $\hat{M}_c(T)$ rather than $\overline{M}_c(T)$ as an assessment of $M_c(T)$.

More generally, recall by (38) $\hat{M}_c(T) < \overline{M}_c(T)$. Thus, when the expected number of distinct B-modes is at least two, (49) suggests $\hat{M}_c(T)$ will be a better assessment of $M_c(T)$ than $\overline{M}_c(T)$ whenever $M_c(T) < \hat{M}_c(T)$.

4.3.6 Example. The following example is taken from [1] and illustrates application of the Crow/AMSAA model.

Example 4.3.6.1

Data were generated by a computer simulation with $\lambda_A = 0.02$, $\lambda_B = 0.1$, $K = 100$ and the d_i's distributed according to a beta distribution with mean 0.7. The simulation portrayed a system tested for $T = 400$ hours. The simulation generated $N = 42$ failures with $N_A = 10$ and $N_B = 32$. The thirty-two B-mode failures were due to M=16 distinct B-modes. The B-modes are labeled by the index i where the first occurrence time for mode i is t_i and $0 < t_1 < t_2 < \cdots < t_{16} < T = 400$.

Table 4.3.6.1 lists, for each B-mode i, the time of first occurrence followed by the times of subsequent occurrences (if any). Column 3 of the table lists N_i, the total number of occurrences of B-mode i during the test period. Column 4 contains the assessed fix effectiveness factors for each of the observed B-modes. Column 5 has the assessed expected number of type i B-modes that would occur in T=400 hours after implementation of the fix. Finally, the last column contains the base e logarithms of the B-mode first occurrence times. These are used to calculate $\hat{\beta}$.

Table 4.3.6.1 Projection Example Data.

B-mode	Failure Times (hrs)	N_i	d_i^*	$(1-d_i^*)N_i$	$\ln t_i$
1	15.04, 254.99	2	.67	.66	2.7107
2	25.26, 120.89, 366.27	3	.72	.84	3.2292
3	47.46, 350.2	2	.77	.46	3.8599
4	53.96, 315.42	2	.77	.46	3.9882
5	56.42, 72.09, 339.97	3	.87	.39	4.0328
6	99.57, 274.71	2	.92	.16	4.6009
7	100.31	1	.50	.50	4.6083
8	111.99, 263.47, 373.03	3	.85	.45	4.7184
9	125.48, 164.66, 303.98	3	.89	.33	4.8321
10	133.43, 177.38, 324.95, 364.63	4	.74	1.04	4.8936
11	192.66	1	.70	.30	5.2609
12	249.15, 324.47	2	.63	.74	5.5181
13	285.01	1	.64	.36	5.6525
14	379.43	1	.72	.28	5.9387
15	388.97	1	.69	.31	5.9635
16	395.25	1	.46	.54	5.9795
Totals		32	11.54	7.82	75.7873

From Equation (1) and Table 4.3.6.1, the adjustment procedure estimate of r(T) = r(400) is

$$\hat{r}_{adj}(400) = \left(\frac{1}{400}\right)\left(N_A + \sum_{i=1}^{16}\left(1 - d_i^*\right)N_i\right)$$

$$= \frac{10 + 7.82}{400} = 0.04455$$

Thus the adjustment procedure estimate of the system MTBF is

$$\left\{\hat{r}_{adj}(400)\right\}^{-1} = \frac{400}{17.82} = 22.45$$

Looking at Equation (40), we can see that the adjustment procedure estimate of system failure intensity after implementation of the fixes is simply $\hat{\rho}_{GP}$, the estimated growth potential failure intensity. Thus

$$\hat{\rho}_{GP} = \hat{r}_{adj}(400) = 0.04455$$

Also, the estimate of the system growth potential MTBF is

$$\hat{\rho}_{GP}^{\;-1} = \left\{\hat{r}_{adj}(400)\right\}^{-1} = 22.45$$

To obtain an estimate with less bias of the system's failure intensity and corresponding MTBF at T=400 hours, after incorporation of fixes to the sixteen surfaced B-modes, we use the Crow/AMSAA model estimation equation (32). This projection is given by

$$\hat{\rho}_c(400) = \hat{\rho}_{GP} + \left(\frac{\hat{\beta}}{400}\right)\sum_{i \in obs} d_i^*$$

$$= 0.04455 + \left(\frac{\hat{\beta}}{400}\right)(11.54) \qquad (50)$$

The mle $\hat{\beta}$ is obtained from Equation (26), i.e.,

$$\hat{\beta} = \frac{m}{\sum_{i=1}^{m} \ln\left(\frac{T}{t_i}\right)} = \frac{m}{m \ln T - \sum_{i=1}^{m} \ln t_i}$$

$$= \frac{16}{16\ln 400 - 75.7873} = 0.7970$$

Thus, by (50), the Crow/AMSAA projection for the system failure intensity, based on $\hat{\beta}$, is

$$\hat{\rho}_c(400) = 0.04455 + \left(\frac{0.7970}{400}\right)(11.54)$$

$$= 0.06754$$

The corresponding MTBF projection is

$$\{\hat{\rho}_c(400)\}^{-1} = 14.81$$

A nearly unbiased assessment of the system failure intensity, for $d_i^* = d_i$, can be obtained by using $\bar{\beta}_m$ instead of $\hat{\beta}$. Recall by (28),

$$\bar{\beta}_m = \left(\frac{m-1}{m}\right)\hat{\beta} = \left(\frac{15}{16}\right)(0.7970) = 0.7472$$

By equation (33), the projected system failure intensity based on $\bar{\beta}_m$ is

$$\bar{\rho}_c(400) = \hat{\rho}_{GP} + \frac{\bar{\beta}_m}{T}\sum_{i\in obs} d_i^*$$

$$= 0.04455 + \left(\frac{0.7472}{400}\right)(11.54)$$

$$= 0.06611$$

The corresponding MTBF projection is

$$\{\bar{\rho}_c(400)\}^{-1} = 15.13$$

As discussed in Section 4.3.5, we recommend basing the projected system failure intensity on $\bar{\rho}_c(T)$ which uses $\bar{\beta}_m$, but assess the projected system MTBF by using $\hat{\beta}$. Thus in this example we would recommend assessing the projected system failure intensity by

$$\overline{\rho}_c(400) \quad = \quad 0.06611$$

and the projected system MTBF by

$$\{\hat{\rho}_c(400)\}^{-1} \quad = \quad 14.81$$

4.4 The AMSAA Maturity Projection Model (AMPM) – Continuous.

4.4.1 Introduction. The continuous version of the AMPM assumes the test duration is measured in a continuous scale such as time or miles. Throughout this section AMPM will refer to the continuous version of the model and we shall refer to time as the measure of test duration.

The AMPM addresses making reliability projections in several situations of interest. One case corresponds to that addressed by the Crow/AMSAA projection model introduced in [1] and discussed in Section 4.3. This is the situation in which all fixes to B-modes are implemented at the end of the current test phase, Phase I, prior to commencing a follow-on test phase, Phase II. The projection problem is to assess the expected system failure intensity at the start of Phase II. Another situation handled by the AMPM estimation procedure is the case where the reliability of the unit under test has been maturing over Test Phase I due to implemented fixes during Phase I. This case includes the situations where

(i) all surfaced B-modes in Test Phase I have fixes implemented within this test phase or

(ii) some of the surfaced B-modes are addressed by fixes within Test Phase I and the remainder are treated as delayed fixes, i.e., are fixed at the conclusion of Test Phase I, prior to commencing Test Phase II.

A third type of projection of interest involves projecting the system failure intensity at a future program milestone. This future milestone may occur beyond the commencement of the follow-on test phase.

All the above type of projections are based on the Phase I B-mode first occurrence times, whether the associated B-mode fix is implemented within the current test phase or delayed (but implemented prior to the projection time). In addition to the B-mode first occurrence times, the projections are based on an average fix effectiveness factor (FEF). This average is with respect to all the potential B-modes, whether surfaced or not. However, as in the Crow/AMSAA model, this average FEF is assessed based on the surfaced B-modes. For the AMPM model, the set of surfaced B-modes would typically be a mixture of B-modes addressed with fixes during the current test phase as well as those addressed beyond the current test phase.

In some instances, a reliability projection for a future milestone can be based on extrapolating a reliability growth tracking curve. Such a curve only utilizes cumulative failure times and does not use B-mode fix effectiveness factors. This is a valid projection approach provided it is reasonable to expect that the observed pattern of reliability growth will continue up through the milestone of interest. However, this pattern could change in a pronounced manner. Reasons for such a change include

(i) a change in the test environment;

(ii) less future resources to analyze and implement effective corrective actions; and

(iii) jumps in reliability due to delayed fixes.

If extrapolating the current tracking curve is not deemed suitable due to considerations such as above, the AMPM projection methodology may be useful. Unlike assessments based on the tracking model, the AMPM assessments are independent of the fix discipline, as long as the fixes are implemented prior to the projection milestone date of interest. Unlike the reliability growth tracking model in [2], the AMPM (as well as the Crow/AMSAA projection model) utilize a non-homogeneous Poisson process with regard to the number of distinct B-modes that occur by test duration t. The associated pattern of B-mode first occurrence times is not dependent on the corrective action strategy, under the assumption that corrective actions are not inducing new B-modes to occur. Thus the AMPM assessment procedure is not upset by jumps in reliability due to delayed groups of fixes. In contrast, reliability growth tracking curve methodology utilizes the pattern of cumulative failure times. Such a pattern is sensitive to the corrective action strategy. Thus a reliability growth tracking curve model may not be appropriate for fitting failure data or for extrapolating due to a corrective action strategy that is not compatible with the model.

Note that AMPM reliability projections for a future milestone would be optimistic if corrective actions beyond the current test phase were less effective than the average FEF assessment based on B-modes surfaced through the current test phase. Also, a change in the future testing environment could result in a new set of potential failure modes or affect the rates of occurrence of the original set of failure modes. Either of these circumstances would tend to degrade the accuracy of the AMPM reliability projection.

Another instance in which a reliability projection model would be useful is when the current test phase contains a number of design configurations of the units under test due to incorporation of reliability fixes during the test phase. If there is a lack of fit of the reliability growth tracking model over these configurations then the tracking model should not be used to assess the reliability of the latest configuration or for extrapolation to a future milestone. Such a lack of fit may be due to the corrective action process, i.e., when the fixes are implemented and their effectivity. As pointed out earlier, the AMPM, unlike a tracking model, is insensitive to any nonsmoothness in the expected number of failures versus test time that results from the timing or effectivity of corrective actions. Thus in such a situation, program management may wish to use a projection method such as the AMPM to assess the reliability of the current configuration or to project the expected reliability at a future milestone.

As discussed in [3], the AMPM can also be used to construct a useful reliability maturity metric. This metric is the fraction of the initial system B-mode failure intensity, λ_B, surfaced by test duration t. By this we mean the expected fraction of λ_B due to B-modes surfaced by t. This concept will be expanded upon in a later subsection.

Prior to presenting the model equations and estimation procedures, we shall list the associated notation and assumptions.

4.4.2 AMPM Notation and Assumptions.

Notation:

α, β Parameters for gamma density function, where $\alpha > -1$ and $\beta > 0$.

$\alpha!$ Denotes the integral $\int_0^\infty x^\alpha \, e^{-x} dx$ for $\alpha > -1$.

Λ Gamma random variable.

$\Gamma(\alpha, \beta)$ Denotes gamma random variable with parameters $\alpha > -1$, $\beta > 0$.

f_Λ Denotes density function for $\Lambda \sim \Gamma(\alpha, \beta)$, where

$$f_\Lambda(\lambda) = \frac{\lambda^\alpha e^{-\frac{\lambda}{\beta}}}{\alpha! \, \beta^{\alpha-1}} \quad \text{for } \lambda > 0;$$

$$= 0 \quad \text{elsewhere}$$

$\underline{\Lambda} = (\Lambda_1, \cdots, \Lambda_K)$ Random sample of size K from $\Gamma(\alpha, \beta)$.

$\underline{\lambda} = (\lambda_1, \cdots, \lambda_K)$ Realization of $\underline{\Lambda}$.

$\lambda_{B,K}$ Expected value of $\sum_{i=1}^{K} \Lambda_i$.

$\lambda_{B,\infty} = \lim_{K \to \infty} \lambda_{B,K}$

$\mu(t\,;\underline{\lambda})$	Expected number of distinct B-modes conditioned on $\underline{\Lambda} = \underline{\lambda}$.
$h(t\,;\underline{\lambda})$	Expected rate of occurrence of B-modes given $\underline{\Lambda} = \underline{\lambda}$.
$h(t)$	Unconditional expected B-mode rate of occurrence.
$r(t\,;\underline{\lambda})$	System failure intensity after fixes to B-modes surfaced by t have been implemented, conditioned on $\underline{\Lambda} = \underline{\lambda}$.
$\rho(t;\underline{\lambda})$	Expected value of $r(t\,;\underline{\lambda})$ with respect to random first occurrence times of B-modes.
$\rho(t)$	Expectation of $\rho(t;\underline{\Lambda})$ with respect to $\underline{\Lambda}$.
$I_i(t)$	Equals 1 if B-mode i occurs by t, equals 0 otherwise.
$u(t;\underline{\lambda})$	Failure intensity at time t due to unsurfaced B-modes, conditioned on $\underline{\Lambda} = \underline{\lambda}$.
$s(t)$	Unconditional expected failure intensity due to set of B-modes surfaced by t, in absence of any fixes.
$\theta(t)$	Fraction of $\lambda_{B,K}$ surfaced as a function of t.
t_i	Time of first occurrence of B-mode i.
$\underline{t} = (t_1, \cdots, t_m)$	
$L(m,\underline{t},\underline{\lambda})$	Likelihood function for the test data $(m,\underline{t})$ given $\underline{\Lambda} = \underline{\lambda}$.
$L(m,\underline{t})$	Expectation of $L(m,\underline{t},\underline{\Lambda})$.
ln	Natural logarithm (base “e”).
Z	$\ln\{m!\,L(m,\underline{t})\}$

ν_K	(α, β, K)
$\hat{\nu}_K$	$\left(\hat{\alpha}_K, \hat{\beta}_K, K\right)$
obs	Set of indices associated with m observed B-modes.
K_0	Greatest lower bound for set of K-values for which AMPM mle's are well defined.
K_{IBM}	IBM model mle of K.

Additional Assumptions for AMPM – Continuous

- The time to first occurrence is exponentially distributed for each failure mode.
- For $i = 1,2,\cdots,K$, the effectiveness of a fix associated with B-mode i is independent of the mode's initial rate of occurrence λ_i.
- The B-mode initial rates of occurrence $(\lambda_1,\cdots,\lambda_K)$ constitute the realization of a random sample $(\Lambda_1,\cdots,\Lambda_K)$ from a gamma distribution with density f_Λ. This models mode-to-mode variation in the B-mode initial failure rates. That is, we assume the Λ_i $(i = 1,\cdots,K)$ are independent and identically distributed (IID) random variables, where $\Lambda_i \sim \Gamma(\alpha,\beta)$.

4.4.3 AMPM Development. The AMPM provides a procedure for assessing the system failure intensity $r(t;\underline{\lambda})$. Recall $r(t;\underline{\lambda})$ denotes the system failure intensity after fixes to all B-modes surfaced by test time t have been implemented.

Note $\underline{\lambda} = (\lambda_1,\cdots,\lambda_K)$ denotes the initial B-mode rates of occurrence. In particular, consider B-mode i. If this mode does not occur by t then its rate of occurrence at t is still λ_i. However, if B-mode i occurs by t then, by our definition of $r(t;\underline{\lambda})$, the contribution of this mode to $r(t;\underline{\lambda})$ is only $(1-d_i)\lambda_i$ due to the implemented fix (or fixes) to mode i by t. We may conveniently mathematically express the contribution of B-mode i to $r(t;\underline{\lambda})$ by

$$\{1 - d_i I_i(t)\}\lambda_i \tag{1}$$

Thus

$$r(t;\underline{\lambda}) = \lambda_A + \sum_{i=1}^{K} \{1 - d_i I_i(t)\} \lambda_i$$

$$= \lambda_A + \sum_{i=1}^{K} \lambda_i - \sum_{i=1}^{K} d_i \lambda_i I_i(t) \qquad (2)$$

As in the Crow/AMSAA model, the AMPM assesses the system failure intensity $r(t;\underline{\lambda})$ by an assessment of the expected value of $r(t;\underline{\lambda})$, i.e. $\rho(t;\underline{\lambda}) = E(r(t;\underline{\lambda}))$. Note by (2) we have

$$\rho(t;\underline{\lambda}) = E(r(t;\underline{\lambda}))$$

$$= \lambda_A + \sum_{i=1}^{K} \lambda_i - \sum_{i=1}^{K} d_i \lambda_i E(I_i(t)) \qquad (3)$$

In Appendix D, Annex 1 we show,

$$E[I_i(t)] = 1 - e^{-\lambda_i t} \qquad (4)$$

where the expectation is with respect to the time of first occurrence of B-mode i. This yields

$$\rho(t;\underline{\lambda}) = \lambda_A + \sum_{i=1}^{K} (1 - d_i) \lambda_i + \sum_{i=1}^{K} d_i \lambda_i e^{-\lambda_i t} \qquad (5)$$

In Section 4.3 (where the argument $\underline{\lambda}$ was suppressed) it was noted that the Crow/AMSAA model approximates $\rho(t;\underline{\lambda})$ by

$$\rho_c(t;\underline{\lambda}) = \lambda_A + \sum_{i=1}^{K} (1 - d_i) \lambda_i + \mu_d \, h_c(t;\underline{\lambda}) \qquad (6)$$

with

$$h_c(t;\underline{\lambda}) = uvt^{v-1} \qquad (7)$$

for positive constants u, v. This form for the expected rate of occurrence of new B-modes corresponds to approximating the expected number of distinct B-modes occurring over [0, t] by

$$\mu_c(t;\underline{\lambda}) = ut^{v} \qquad (8)$$

Recall the Crow/AMSAA procedure estimates the constants u, v by the mle statistics based on the B-mode first occurrence times. The summation term in (6) is assessed as

$$\sum_{i \in obs} \left(1 - d_i^*\right) \frac{N_i}{t} \tag{9}$$

where d_i^* is the assessed fix effectiveness factor for observed B-mode i, and N_i is the number of occurrences of failures during [0,t] attributed to B-mode i. Note in the Crow/AMSAA procedure all fixes are assumed to be delayed to the end of the period [0,t]. Under this assumption $\frac{N_i}{t}$ is an unbiased estimate of λ_i. However, if fixes to B-modes are implemented prior to the end of this period (9) may not be an adequate assessment of $\sum_{i=1}^{K} \left(1 - d_i\right) \lambda_i$.

The AMPM does not attempt to assess $\rho\left(t; \underline{\lambda}\right)$ by estimating each λ_i. Instead the AMPM approach is to view $\left(\lambda_1, \cdots, \lambda_K\right)$ as a realization of a random sample $\underline{\Lambda} = \left(\Lambda_1, \cdots, \Lambda_K\right)$ from the gamma random variable $\Gamma(\alpha, \beta)$. This allows one to utilize all the B-mode times to first occurrence to estimate the gamma parameters α, β. Thus in place of directly assessing $\rho\left(t; \underline{\lambda}\right)$, the AMPM uses estimates of α and β to assess the expected value of $\rho\left(t; \underline{\Lambda}\right)$ where

$$\rho\left(t; \underline{\Lambda}\right) \;=\; \lambda_A + \sum_{i=1}^{K} \left(1 - d_i\right) \Lambda_i + \sum_{i=1}^{K} d_i \Lambda_i e^{-\Lambda_i t} \tag{10}$$

This assessed value is then taken as the AMPM assessment of the system failure intensity after fixes to all B-modes surfaced over [0,t] have been implemented. This approach does away with the need to estimate individual λ_i. Trying to adequately estimate individual λ_i could be particularly difficult in the case where many fixes are implemented prior to the end of the period [0,t].

From Equation (10) we see that the expected value of $\rho\left(t; \underline{\Lambda}\right)$ with respect to the random sample $\underline{\Lambda}$, denoted by $\rho(t)$, is given by

$$\rho(t) \;=\; \lambda_A + \sum_{i=1}^{K} \left(1 - d_i\right) E\left(\Lambda_i\right) + \sum_{i=1}^{K} d_i \, E\left(\Lambda_i e^{-\Lambda_i t}\right)$$

(11)

Recall the Λ_i are IID with $\Lambda_i \sim \Lambda$. Thus $E(\Lambda_i) = E(\Lambda)$ and $E\left(\Lambda_i e^{-\Lambda_i t}\right) = E\left(\Lambda e^{-\Lambda t}\right)$ for $i = 1, \cdots, K$. After rearranging terms and replacing $\sum_{i=1}^{K} d_i$ by $K\,\mu_d$, $E(\Lambda_i)$ by $E(\Lambda)$, and $E\left(\Lambda_i e^{-\Lambda_i t}\right)$ by $E\left(\Lambda e^{-\Lambda t}\right)$ we arrive at

$$\rho(t) \quad = \quad \lambda_A + (1-\mu_d)\{K\,E(\Lambda)\} + \mu_d\left\{K\,E\left(\Lambda\,e^{-\Lambda t}\right)\right\} \tag{12}$$

Next note

$$\lambda_{B,K} \quad = \quad E\left(\sum_{i=1}^{K} \Lambda_i\right) \quad = \quad K\,E(\Lambda) \tag{13}$$

Thus we can express $\rho(t)$ by

$$\rho(t) \quad = \quad \lambda_A + (1-\mu_d)\lambda_{B,K} + \mu_d\left\{K\,E\left(\Lambda e^{-\Lambda t}\right)\right\} \tag{14}$$

To interpret the term $K\,E\left(\Lambda e^{-\Lambda t}\right)$ in (14) we first note that in Appendix D, Annex 1, it is shown that

$$\mu(t;\underline{\lambda}) \quad = \quad \sum_{i=1}^{K}\left(1-e^{-\lambda_i t}\right)$$

Thus the expected rate of occurrence of new B-modes at t, given $\underline{\lambda}$, is

$$h(t;\underline{\lambda}) \quad = \quad \frac{d\,\mu(t;\underline{\lambda})}{dt} \quad = \quad \sum_{i=1}^{K} \lambda_i e^{-\lambda_i t}$$

Consider the average (i.e. expected) value of $h(t;\underline{\Lambda}) = \sum_{i=1}^{K} \Lambda_i e^{-\Lambda_i t}$ over all possible random samples $\underline{\Lambda} = (\Lambda_1, \cdots, \Lambda_K)$, where $\Lambda_i \sim \Lambda$ for $i = 1, \cdots, K$. We obtain

$$E(h(t;\Lambda)) \quad = \quad \sum_{i=1}^{K} E\left(\Lambda_i e^{-\Lambda_i t}\right) \quad = \quad K\,E\left(\Lambda e^{-\Lambda t}\right) \tag{15}$$

Let $h(t) = E(h(t;\Lambda))$. Thus $h(t)$ is the unconditional expected rate of occurrence of new B-modes at test time t averaged over all possible random samples $\underline{\Lambda}$. By (14) and (15) we have

$$\rho(t) \quad = \quad \lambda_A + (1-\mu_d)\lambda_{B,K} + \mu_d h(t) \tag{16}$$

This expression for $\rho(t)$ is similar in form to the Crow/AMSAA approximation to $\rho(t;\underline{\lambda})$ given in Equation (14):

$$\rho_c(t;\underline{\lambda}) \quad = \quad \lambda_A + \sum_{i=1}^{K}(1-d_i)\lambda_i + \mu_d h_c(t)$$

where reference to $\underline{\lambda}$ was suppressed in the notation.

The expression in (16) for the expected system failure intensity after incorporation of B-mode fixes is actually quite appealing to one's intuition if put in a slightly different form. To arrive at this form we shall simply subtract and add the term $h(t)$ on the right hand side of Equation (16). Doing this we can express $\rho(t)$ by

$$\rho(t) \quad = \quad \lambda_A + (1-\mu_d)\{\lambda_{B,K} - h(t)\} + h(t) \qquad (17)$$

Now we see that $\rho(t)$ is the sum of three failure intensities. The first is simply the constant failure intensity due to the A-modes. To consider the second failure intensity we shall first consider $h(t)$. We have shown that this term is the expected rate of occurrence of new B-modes at test time t averaged over the random samples $\underline{\Lambda}$. Additionally, $h(t)$ is the expected failure intensity contribution to $\rho(t)$ due to the set of B-modes that have not been surfaced by t. To see this, first note that the failure intensity at time t, conditioned on $\underline{\Lambda} = \underline{\lambda}$, due to unsurfaced B-modes is $u(t;\underline{\lambda})$ where

$$u(t;\underline{\lambda}) \quad = \quad \sum_{i=1}^{K}\{1 - I_i(t)\}\lambda_i \qquad (18)$$

Recall by (4),

$$E[I_i(t)] \quad = \quad 1 - e^{-\lambda_i t}$$

with respect to the first occurrence of B-mode i. Thus by (18) we have

$$E[u(t;\underline{\lambda})] \quad = \quad \sum_{i=1}^{K}\lambda_i - \sum_{i=1}^{K}\lambda_i E[I_i(t)]$$

$$= \quad \sum_{i=1}^{K}\lambda_i e^{-\lambda_i t} \quad = \quad h(t;\underline{\lambda}) \qquad (19)$$

It immediately follows from (19) that $h(t)$ is the unconditional expected failure intensity due to the set of unsurfaced B-modes at time t, since $h(t) = E(h(t;\underline{\Lambda}))$.

Finally, we consider the second term of $\rho(t)$ in (17). In the absence of any fixes, the sum of $h(t)$ and the unconditional expected failure intensity due to the set of B-modes surfaced by t, denoted by s(t), must equal $\lambda_{B,K}$. Thus $s(t)=\lambda_{B,K}-h(t)$. If we implement fixes to the B-modes surfaced by t with an average FEF equal to d, then the residual expected failure intensity due to the set of surfaced B-modes would be

$$(1-d)s(t) \quad = \quad (1-d)\{\lambda_{B,K}-h(t)\} \tag{20}$$

In the above equations we can replace $\lambda_{B,K}$ by h(0) since at t=0 all B-modes are unsurfaced. Thus

$$h(0) \quad = \quad \lambda_{B,K} \tag{21}$$

As in Section 4.3, we call the residual expected failure intensity approached by $\rho(t)$ as t tends towards infinity the growth potential failure intensity, denoted by ρ_{GP}. Since $\lim_{t\to\infty} h(t)=0$ we have

$$\rho_{GP} \quad = \quad \lambda_A+(1-\mu_d)\lambda_B \tag{22}$$

Note this expression has the same form as that for the growth potential in the Crow/ AMSAA model. The quantity ρ_{GP}^{-1} is called the growth potential MTBF. The growth potential for the AMPM is used in the same way as indicated in Section 4.3 for the Crow/ AMSAA model.

Another useful quantity is the fraction of the system expected initial B-mode failure intensity, λ_B, surfaced as a function of test time t. We shall let $\theta(t)$ denote this quantity. Thus, by definition of s(t), we have

$$\theta(t) \quad = \quad \frac{s(t)}{\lambda_B} \quad = \quad \frac{\lambda_B-h(t)}{\lambda_B} \tag{23}$$

Note that $\theta(t)$ is independent of the corrective action process. By this we mean that $\theta(t)$ does not depend on when fixes are implemented nor on how effective they are.

The function $\theta(t)$ can usefully serve as a measure of system maturity. Observe that for a test of duration t, no matter how effective our fixes are, we can only eliminate at most a fraction equal to $\theta(t)$ of the B-mode contribution to the initial system failure intensity. Thus low values of $\theta(t)$ would indicate additional testing is required to surface a set of B-modes that account for a significant part of λ_B. A high value for $\theta(t)$ could indicate that further testing is not cost effective. Resources would be better expended toward formulating and implementing corrective actions for the surfaced B-modes. As part of a reliability growth plan it would be useful to specify goals for $\theta(t)$ at several program milestones.

Next we shall express the key AMPM reliability projection quantities in terms of K and the gamma parameters α and β. By Appendix D, Annex 2, we have

$$\lambda_{B,K} \quad = \quad K\beta(\alpha+1) \tag{24}$$

$$\mu(t) \quad = \quad K\left\{1-(1+\beta t)^{-(\alpha+1)}\right\} \tag{25}$$

$$h(t) \quad = \quad \frac{K\beta(\alpha+1)}{(1+\beta t)^{\alpha+2}} \quad = \quad \frac{d\mu(t)}{d t} \tag{26}$$

$$\rho(t) \quad = \quad \lambda_A + (1-\mu_d)K\beta(\alpha+1) + \frac{\mu_d K\beta(\alpha+1)}{(1+\beta t)^{\alpha+2}} \tag{27}$$

and

$$\theta(t) \quad = \quad 1-(1+\beta t)^{-(\alpha+2)} \tag{28}$$

Utilizing equation (24) for $\lambda_{B,K}$ we can also express $h(t)$ and $\rho(t)$ as follows:

$$h(t) \quad = \quad \frac{\lambda_{B,K}}{(1+\beta t)^{\alpha+2}} \tag{29}$$

and

$$\rho(t) \quad = \quad \lambda_A + (1-\mu_d)\lambda_{B,K} + \frac{\mu_d\,\lambda_{B,K}}{(1+\beta t)^{\alpha+2}} \tag{30}$$

In the next section, we shall consider the behavior of the AMPM as K increases. Limiting expressions for the AMPM quantities in (24) through (30) will be obtained as $K \to \infty$ under natural assumptions about $\lambda_{B,K}$ and β_K. Then parameter estimation procedures will be specified for the finite K AMPM and the limiting parameters as $K \to \infty$.

4.4.4 Limiting Behavior of AMPM. We shall now consider the limiting behavior of the AMPM as K increases. To do so we first define step processes $\{X_{K,i}(t), 0 \le t < \infty\}$ *for* $i = 1,\cdots,K$ where

$$X_{K,i}(t) \quad = \quad \begin{cases} 1 & \textit{if } B-\text{mode } i \text{ occurs by t} \\ 0 & \text{otherwise} \end{cases}$$

Note

$$\Pr(X_{K,i}(t) \geq 2) \quad = \quad 0 \tag{31}$$

and

$$\Pr(X_{K,i}(t) = 1) \quad = \quad 1 - \Pr(X_{K,i}(t) = 0) \tag{32}$$

Thus to complete our definition of these processes, we need only specify $\Pr(X_{K,i}(t) = 0)$. To keep the definition of these processes consistent with the AMPM assumptions we define

$$\Pr(X_{K,i}(t) = 0) \quad = \quad \int_0^\infty e^{-\lambda t} f_\Lambda(\lambda) d\lambda \tag{33}$$

where $\Lambda \sim \Gamma(\alpha, \beta)$ and f_Λ is the previously defined gamma density function with $\alpha = \alpha_K$ and $\beta = \beta_K$. Note $X_{K,i}(t)$ is the unconditional AMPM indicator function for B-mode i corresponding to the earlier defined conditional indicator function $I_i(t)$ where

$$\Pr(I_i(t) = 0) \quad = \quad e^{-\lambda_i t}$$

and subscript K was suppressed. Note by (33) and Appendix D, Annex 2

$$\Pr(X_{K,i}(t) = 0) \quad = \quad E(e^{-t\Lambda}) \quad = \quad \Psi(-t) \quad = \quad (1 + \beta_K t)^{-(\alpha+1)} \tag{34}$$

By (32) and (34) we obtain

$$\mu_K(t) \quad \underline{\Delta} \quad E\left[\sum_{i=1}^{K} X_{K,i}(t)\right]$$

$$= \quad \sum_{i=1}^{K} \Pr(X_{K,i}(t) = 1) \quad = \quad K - K(1 + \beta_K t)^{-(\alpha+1)} \quad = \quad \mu(t) \tag{35}$$

for $(\alpha_K, \beta_K) = (\alpha, \beta)$. Thus the AMPM step processes $\{X_{K,i}(t), 0 \leq t < \infty\}, \quad 1 \leq i \leq K$, give rise to our previously developed AMPM.

To investigate the behavior of our projection model as K increases, we must specify the limiting behavior of α_K and β_K. Since β_K is simply a scale factor for test time t it is reasonable to keep β_K fixed, say $\beta_K = \beta \in (0,\infty)$. Recall by (24), $\lambda_{B,K} = K\ \beta(\alpha+1)$. Regardless of the value of K, $\lambda_{B,K}$ represents the unconditional expected B-mode contribution to the initial system failure intensity. Thus it is natural to let $K\ \beta_K(\alpha_K+1) \ = \ \lambda_{B,\infty} \in (0,\infty)$ for all K. Actually, to obtain our results for the limiting behavior of the AMPM we need only insist that

$$\lim_{K\to\infty} \beta_K \quad = \quad \beta_\infty \in (0,\infty) \tag{36}$$

and

$$\lim_{K\to\infty} K\ \beta_K(\alpha_K+1) \quad = \quad \lambda_{B,\infty} \in (0,\infty) \tag{37}$$

We shall simply denote β_∞ and $\lambda_{B,\infty}$ by β and λ_B, respectively. Since $\alpha_K + 1 \geq 0$, (36) and (37) imply

$$\lim_{K\to\infty} \alpha_K \quad = \quad -1 \tag{38}$$

Let $X_K(t)$ be the supposition of the independent step processes $X_{K,i}(t)$, i.e.

$$X_K(t) \ \underset{=}{\Delta} \ \sum_{i=1}^{K} X_{K,i}(t) \tag{39}$$

It is demonstrated in [4] that the stochastic process $\{X_K(t),\ 0 \leq t < \infty\}$ converges to a nonhomogeneous Poisson process (NHPP) with mean value function $\mu_\infty(t)$ as $K \to \infty$, where

$$\mu_\infty(t) \quad = \quad \left(\frac{\lambda_B}{\beta}\right)\ln(1+\beta\, t) \tag{40}$$

This result suggests that for complex systems or subsystems, we can expect our AMPM process $\{X_K(t),\ 0 \leq t < \infty\}$ to behave like a NHPP $\{X_\infty(t),\ 0 \leq t < \infty\}$ where $X_\infty(t)$ is the number of distinct B-modes that occur by t and $E\{X_\infty(t)\} = \mu_\infty(t)$ given in (40).

We can now relate the key AMPM reliability projection quantities in (24) through (28) which depend on K to the corresponding NHPP quantities. To do so we shall subscript the AMPM quantities by K and the NHPP quantities by ∞. Thus, for example, by (24) and limit condition (37) we have

$$\lim_{K\to\infty} \lambda_{B,K} \quad = \quad \lambda_{B,\infty} \in (0,\infty) \tag{41}$$

(where we also denote $\lambda_{B,\infty}$ simply by λ_B). By (26) we also have

$$\mu_K(t) = \int_0^t \frac{K\,\beta_K\,(\alpha_K+1)}{(1+\beta_K z)^{\alpha_K+2}}\,dz$$

Thus

$$\lim_{K\to\infty} \mu_K(t) = \int_0^t \left[\lim_{K\to\infty} \left\{ \frac{K\,\beta_K\,(\alpha_K+1)}{(1+\beta_K z)^{\alpha_K+2}} \right\} \right] dz$$

By (36) through (38) and (40) this yields

$$\lim_{K\to\infty} \mu_K(t) = \int_0^t \frac{\lambda_B dz}{1+\beta z} = \left(\frac{\lambda_B}{\beta}\right) \ln(1+\beta t) = \mu_\infty(t) \tag{42}$$

Again by (26), (36) through (38) and (40), we obtain

$$\lim_{K\to\infty} h_K(t) = \lim_{K\to\infty} \frac{K\,\beta_K(\alpha_K+1)}{(1+\beta_K t)^{\alpha_K+2}}$$

$$= \frac{\lambda_B}{1+\beta t} = \frac{d\mu_\infty(t)}{dt} = h_\infty(t) \tag{43}$$

By (27), (36) through (38) and (43) we arrive at

$$\lim_{K\to\infty} \rho_K(t) = \lim_{K\to\infty} \left\{ \lambda_A + (1-\mu_d) K\,\beta_K\,(\alpha_K+1) + \frac{\mu_d\,K\,\beta_K(\alpha_K+1)}{(1+\beta_K\,t)^{\alpha_K+2}} \right\}$$

$$= \lambda_A + (1-\mu_d)\lambda_B + \frac{\mu_d\,\lambda_B}{1+\beta t} = \lambda_A + (1-\mu_d)\lambda_B + \mu_d\,h_\infty(t)$$

$$= \rho_\infty(t) \tag{44}$$

Additionally, by (22) and (41) we have

$$\lim_{K\to\infty} \rho_{GP,K} = \lim_{K\to\infty} \{\lambda_A + (1-\mu_d)\lambda_{B,K}\}$$

$$= \lambda_A + (1-\mu_d)\lambda_B = \rho_{GP,\infty}$$

(45)

Finally, by (28), (36) and (38) we deduce

$$\lim_{K\to\infty}\theta_K(t) \quad = \quad \lim_{K\to\infty}\left\{1-\left(1+\beta_K\,t\right)^{-(\alpha_K+2)}\right\} \quad = \quad \frac{\beta\,t}{1+\beta\,t}$$

Thus by (43) we conclude

$$\lim_{K\to\infty}\theta_K(t) \quad = \quad \frac{\beta\,t}{1+\beta\,t} \quad = \quad \frac{\lambda_B-h_\infty(t)}{\lambda_B} \quad = \quad \theta_\infty(t) \qquad (46)$$

4.4.5 Estimation Procedure for AMPM. In this section we shall specify the procedures to estimate key AMPM parameters and reliability measures expressed in terms of these parameters. Estimation equations will be given for the finite K and NHPP variants of the continuous AMPM. The model parameter estimators are mle's. Statistical details and further discussion of the estimation procedures are provided in Appendix D, Annex 3.

Our parameter estimates are written in terms of the following data: m = number of distinct B-modes that occur over a test period of length T, $\underline{t}=(t_1,\cdots,t_m)$ where $0<t_1\le t_2\le\cdots\le t_m\le T$ are the first occurrence times of the m observed B-modes, and n_A = number of A-mode failures that occur over test period T. We shall denote an estimate of a model parameter or expression by placing the symbol "^" over the quantity.

The finite K AMPM estimates are based on a specified value of K. If we hold the test data constant and let $K\to\infty$ we obtain AMPM projection estimates that are appropriate for complex subsystems or systems that typically have many potential B-modes. The AMPM limit estimating equations are derived in Appendix D, Annex 3. These equations can also be obtained from mle equations for the NHPP associated with the AMPM. This process was discussed in Section 4.4.4 and has the mean value function given by Equation (40).

Recall α_K,β_K are the gamma parameters for the AMPM where it is assumed the K initial B-mode failure rates are realized values of a random sample from a gamma random variable $\Gamma(\alpha_K,\beta_K)$.

The mle for β_K is $\hat{\beta}_K$ where

$$K = \frac{\left(\sum_{i=1}^{m} \ln \frac{1+\hat{\beta}_K T}{1+\hat{\beta}_K t_i}\right)\left(\sum_{i=1}^{m} \frac{1}{1+\hat{\beta}_K t_i}\right) - \left(\frac{m\hat{\beta}_K}{1+\hat{\beta}_K T}\right)\sum_{i=1}^{m} \frac{T-t_i}{1+\hat{\beta}_K t_i}}{\left\{\ln\left(1+\hat{\beta}_K T\right)\right\}\left(\sum_{i=1}^{m} \frac{1}{1+\hat{\beta}_K t_i}\right) - \left(\frac{m\hat{\beta}_K}{1+\hat{\beta} T}\right) T} \tag{47}$$

The mle for α_K is $\hat{\alpha}_K$ where $\hat{\alpha}_K$ can be easily obtained from $\hat{\beta}_K$ and either equation below. These equations are the maximum likelihood equations for α_K and β_K respectively (see Appendix D, Annex 3):

$$\left(\hat{\alpha}_K + 1\right)^{-1} = m^{-1}\left[K \ln\left(1+\hat{\beta}_K T\right) - \sum_{i=1}^{m} \ln\left(\frac{1+\hat{\beta}_K T}{1+\hat{\beta}_K t_i}\right)\right] \tag{48}$$

$$\hat{\alpha}_K + 1 = \frac{\frac{m}{\hat{\beta}_K} - \sum_{i=1}^{m} \frac{t_i}{1+\hat{\beta}_K t_i}}{\frac{(K-m)T}{1+\hat{\beta}_K T} + \sum_{i=1}^{m} \frac{t_i}{1+\hat{\beta}_K t_i}} \tag{49}$$

Using $\left(\hat{\alpha}_K, \hat{\beta}_K, K\right)$ we can estimate all our finite K AMPM quantities where the A-mode failure rate λ_A is estimated by $\hat{\lambda}_A = {n_A}/{T}$ and the average B-mode fix effectiveness factor μ_d is assessed as

$$\mu_d^* = \frac{1}{m} \sum_{i \in obs} d_i^* \tag{50}$$

In (4.50), the assessment d_i^* of the fix effectiveness factor (FEF) for observed B-mode i will often be based largely on engineering judgement. The value of d_i^* should reflect several considerations: (1) How certain we are that the problem has been correctly identified; (2) the nature of the fix, e.g., its complexity; (3) past FEF experience and (4) any germane testing (including assembly level testing).

Note the left-hand side of Equation (4.47) requires a value for K before we can numerically solve for $\hat{\beta}_K$. In practice we do not know the value of K. We could attempt to use the data $(m,\underline{t})$ to statistically estimate K. However, graphs presented in the next section illustrate the difficulty in obtaining a reasonable estimate for K even for a large data set that appears to fit the model well. Thus we prefer to take the point of view that we should not attempt to statistically assess K. However, by conducting a standard failure modes and effects criticality analysis (FMECA), we can place a lower bound on K, say K_ℓ. Our experience with the AMPM indicates that if K is substantially higher than m, say, e.g., $K \geq 10m$, then our AMPM projection quantities will be insensitive to the value of K. We believe for a complex system or subsystem it will often be the case that $K_\ell \geq 10m$ or at least the unknown value of K will be $10m$ or higher. The factor 10 may be larger than necessary. We suggest exercising the finite K AMPM with several plausible lower bound values for K and comparing the associated projections with those obtained in the limit as $K \to \infty$. This is illustrated for a data set in the next section.

To obtain the limiting AMPM projection model estimates consider the sequence of finite K AMPM estimates $\left\langle \hat{\beta}_K \right\rangle_{K \geq K_0}$ where we assume $\hat{\beta}_K$ satisfies Equation (47) for each $K \geq K_0$. In Appendix D, Annex 3 it is shown that

$$\hat{\beta}_\infty = \lim_{K\to\infty} \hat{\beta}_K \in (0,\infty) \tag{51}$$

is a finite positive value. Moreover, it is demonstrated that

$$\left\{ \ln\left(1+\hat{\beta}_\infty T\right)\right\} \sum_{i=1}^{m} \frac{1}{1+\hat{\beta}_\infty t_i} - \frac{m\hat{\beta}_\infty T}{1+\hat{\beta}_\infty T} = 0 \tag{52}$$

It is also shown that

$$\hat{\alpha}_\infty = \lim_{K\to\infty} \hat{\alpha}_K = -1 \tag{53}$$

where for each $\hat{\beta}_K$, $K \geq K_0$, $\hat{\alpha}_K$ satisfies Equation (48) (or Equation (49)). The limiting AMPM estimates $\hat{\beta}_\infty$ and $\hat{\lambda}_\infty$, given below in Equation (55), can be shown to be mle's for parameters β_∞ and $\lambda_{B,\infty}$. Recall these parameters define the NHPP discussed in Section 4.4 whose mean value function is given in Equation (40).

For ease of reference, the finite K AMPM and limiting AMPM estimates for key projection model quantities are listed below and indexed by K and ∞, respectively:

$$\hat{\lambda}_K = K\hat{\beta}_K\left(\hat{\alpha}_K + 1\right) \tag{54}$$

$$\hat{\lambda}_\infty = \frac{m\hat{\beta}_\infty}{\ln\left(1+\hat{\beta}_\infty T\right)} \tag{55}$$

$$\hat{\mu}_K(t) = K\left[1-\left(1+\hat{\beta}_K t\right)^{-\left(\hat{\alpha}_K+1\right)}\right] \tag{56}$$

$$\hat{\mu}_\infty(t) = \left(\frac{\hat{\lambda}_{B,\infty}}{\hat{\beta}_\infty}\right)\ln\left(1+\hat{\beta}_\infty t\right) \tag{57}$$

$$\hat{h}_K(t) = \frac{\hat{\lambda}_K}{\left(1+\hat{\beta}_K t\right)^{\hat{\alpha}_K+2}} \tag{58}$$

$$\hat{h}_\infty(t) = \frac{\hat{\lambda}_{B,\infty}}{1+\hat{\beta}_\infty t} \tag{59}$$

$$\hat{\rho}_{GP,K} = \hat{\lambda}_A + \left(1-\mu_d^*\right)\hat{\lambda}_{B,K} \tag{60}$$

$$\hat{\rho}_{GP,\infty} = \hat{\lambda}_A + \left(1-\mu_d^*\right)\hat{\lambda}_{B,\infty} \tag{61}$$

$$\hat{\rho}_K(t) = \hat{\rho}_{GP,K} + \mu_d^*\,\hat{h}_K(t) \tag{62}$$

$$\hat{\rho}_\infty(t) = \hat{\rho}_{GP,\infty} + \mu_d^*\,\hat{h}_\infty(t) \tag{63}$$

$$\hat{\theta}_K(t) = 1-\left(1+\hat{\beta}_K t\right)^{-\left(\hat{\alpha}_K+2\right)} \tag{64}$$

$$\hat{\theta}_\infty(t) = \frac{\hat{\beta}_\infty t}{1+\hat{\beta}_\infty t} \tag{65}$$

Note (55) together with (57) imply

$$\hat{\mu}_{\infty}(T) \quad = \quad \left(\frac{\hat{\lambda}_{B,\infty}}{\hat{\beta}_{\infty}}\right)\ln\left(1+\hat{\beta}_{\infty}T\right) \quad = \quad m \tag{66}$$

This agrees with intuition in the sense that $\hat{\mu}_{\infty}(T)$ is an estimate of the expected number of distinct B-modes generated over the test period [0,T] while m is the observed number of distinct B-modes that occur.

Suppose we adopt the view that our "model of reality" for a system or subsystem is the AMPM for a finite K which is large but unknown. Then we can consider the limiting AMPM projection estimates as approximations to the AMPM estimates that correspond to the "true" value of K. Our discussion in this section suggests that over the projection range of $t \geq T$ values of practical interest, the limiting estimates should be good approximations for complex systems or subsystems. In this sense, knowing the "true" value of K is usually unimportant. Note, however, it is useful to have available the computational formulas for the finite K AMPM projection estimators as a function of K. For example, we can compare the graphs of a projection estimator such as $\hat{\rho}_K(t)$ or $\hat{\mu}_K(t)$ over the t range of interest for different values of K to the corresponding limiting estimator. In this fashion we can discern the nature of the convergence, for example, the rapidity of convergence and whether the convergence is strictly increasing or decreasing for t values of interest. This type of graphical analysis is illustrated with an example.

4.4.6 An Example. We shall illustrate several key features of our projection model and associated estimators by applying the model to a data set generated during an Army system development program. Here, we shall just focus on the B-modes and let $\hat{\lambda}_A = 0$. This test data set consists of $m = 163$ B-mode first occurrence times generated over $T = 8000$ "equivalent" mission hours.

In Figure 1, we display the cumulative number of distinct B-modes versus the mission hours. We also display the graphs of $\hat{\mu}_K(t)$ for several values of K. We can show that the greatest lower bound, K_0, for the set of K-values for which the AMPM estimators are well defined corresponds to a degenerate gamma. This limiting gamma density has zero variance and mean equal to λ, where $\lambda_i = \lambda$ for $i = 1, \cdots, K$. To avoid numerical instability, separate maximum likelihood equations were derived and used for this limiting case. On our graphs we have labeled the curves associated with this case (i.e., $K = K_0$) IBM to indicate that this limiting form for $\mu(t)$ coincides with the IBM model [5]. More explicitly, the IBM model uses this $\mu(t)$ for the expected number of "non-random" failures experienced in t test hours. This limiting form for $\mu(t)$ also is used by Musa in his software reliability basic execution time model [6]. It is interesting to note that the opposite AMPM limiting form, $\mu_\infty(t)$, is used by Musa and Okumoto in their Logarithmic Poisson software reliability execution time model[7]. In both of Musa's models, $\mu(t)$ represents the expected number of software failures experienced over test period [0,t], where t denotes execution time.

Note over the data range, i.e., $0 \le t \le 8000$ hours, the graphs of $\hat{\mu}_K(t)$ are visually indistinguishable for $K_{IBM} \le K \le \infty$. In such circumstances the value of K cannot be reasonably assessed from the test data even if one can formally obtain an mle for K. In fact, applying the IBM model (where all λ_i are implicitly assumed to be equal), we always can obtain an mle for K whenever

$$\frac{1}{m}\sum_{i=1}^{m} t_i < \frac{T}{2}$$

(see Musa, Iannino, and Okumoto with respect to the exponential class family[8]). However, it has been our experience that the IBM estimate of K, $K_{IBM} = K_0$, is often only marginally higher than m, the observed number of distinct B-modes. Since $\hat{\mu}_K(t)$ approaches K as $t \to \infty$, such a low estimate of K forces the slope of $\hat{\mu}_K(t)$ to quickly approach zero beyond T (Figure 2). Note $\hat{h}_K(t)$ is the slope of $\hat{\mu}_K(t)$. Thus we can see that such a low estimate of K also quickly forces $\hat{h}_K(t)$ close to zero for $t \ge T$. This in turn tends to produce an "optimistic" failure intensity projection, especially when the assessed value of d is high. This follows from the formula

$$\hat{\rho}_K(t) \quad = \quad \hat{\lambda}_A + \left(1 - \overset{*}{\mu}_d\right)\hat{\lambda}_{B,K} + \overset{*}{\mu}_d\, \hat{h}_K(t) \tag{26}$$

which, by Equations (24) and (25), applies for $K_{IBM} \le K \le \infty$. Thus a good fit over [0,T] is not a sufficient condition to ensure that a projection model will provide reasonable projection estimates for $t \ge T$.

Looking at Figure 3, as one might expect, the model with $K = \infty$ appears to provide a more conservative estimate of $\rho_K^{-1}(t)$ for $t \ge T$ than do the finite K estimators. However, for $t \ge T$, it is important to note that the $\hat{\mu}_K(t)$, $\hat{\rho}_K^{-1}(t)$ and $\hat{\theta}_K(t)$ graphs, displayed in Figures 1 and 2, 3, and 4, respectively, quickly become much closer to the corresponding $K = \infty$ graph than to the $K = K_{IBM}$ graph as K increases above K_{IBM}.

Observe from Figure 4, $\hat{\theta}_K(8000) \approx .67$ for $K_{IBM} \le K \le \infty$. Thus, whatever the "true" value of K, we estimate that the remaining B-modes contribute about

$.33\lambda_B$ **to the system failure intensity.**

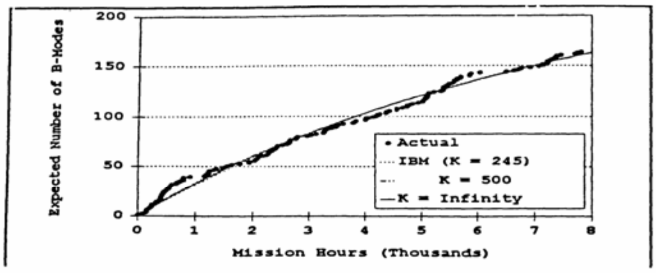

Figure 1. Expected vs Actual Number of B-Modes.

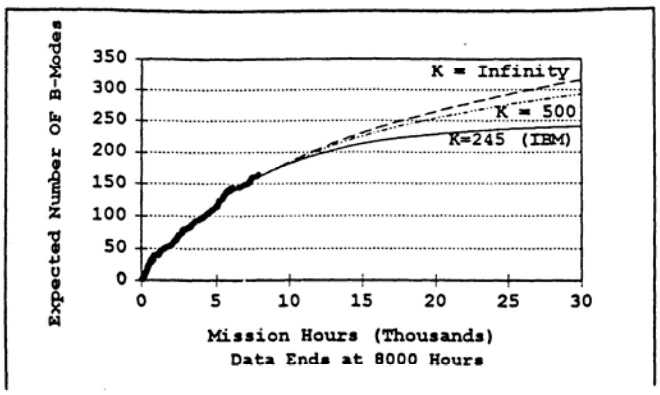

Figure 2. K=245 (IBM) vs K=Infinity.

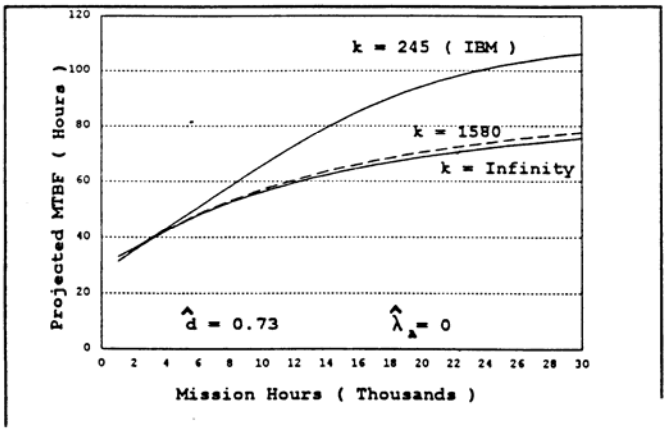

Figure 3. Projected MTBF.

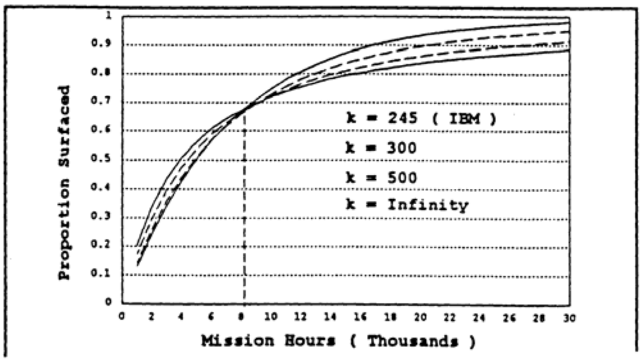

Figure 4. Proportion of Initial B-Mode Intensity Surfaced.

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APPENDIX D

This appendix utilizes the notation in Section 4.4.

Annex 1

We shall show the following:

(1) $E[I_i(t)] \;=\; 1 - e^{-\lambda_i t}$

(2) $\mu(t;\underline{\lambda}) \;=\; \sum_{i=1}^{K}\left(1 - e^{-\lambda_i t}\right)$

(3) $h(t;\underline{\lambda}) \;=\; \sum_{i=1}^{K}\lambda_i e^{-\lambda_i t}$

To show (1) observe that $I_i(t)$ is a random variable that only takes on the values zero and one. Thus

$$E[I_i(t)] \;=\; (0)\Pr(I_i(t)=0)+(1)\Pr(I_i(t)=1)$$

$$=\; \Pr(I_i(t)=1) \;=\; 1 - e^{-\lambda_i t}$$

To show (2), let $M(t)$ denote the number of distinct B-modes that occur by t. Then

$$M(t) \;=\; \sum_{i=1}^{K} I_i(t)$$

Thus

$$\mu(t;\underline{\lambda}) \;=\; E(M(t)) \;=\; \sum_{i=1}^{K} E[I_i(t)] \;=\; \sum_{i=1}^{K}\left(1 - e^{-\lambda_i t}\right)$$

Note (3) follows from (2) since

$$h(t;\underline{\lambda}) \;=\; \frac{d\,\mu(t;\underline{\lambda})}{d\,t} \;=\; \sum_{i=1}^{K}\lambda_i e^{-\lambda_i t}$$

Annex 2

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Recall $\Lambda \sim \Gamma(\alpha, \beta)$. Let Ψ denote the moment generating function for Λ. Thus, by definition, $\Psi(x) = E(e^{x\Lambda})$ for all real x for which the expectation with respect to Λ exists. One can show that Ψ is defined for $x < \dfrac{1}{\beta}$ and $\Psi(x) = (1 - \beta x)^{-(\alpha+1)}$ (see e.g. Mood and Graybill[9]). We shall utilize $\Psi(x)$ to express $\lambda_{B,K}$, $\mu(t)$, $h(t)$, $\rho(t)$, and $\theta(t)$ in terms of K and the gamma parameters α and β. We summarize our results below:

(1) $\lambda_{B,K} \quad = \quad K\,\beta(\alpha + 1)$

(2) $\mu(t) \quad = \quad K\left[1 - (1 + \beta t)^{-(\alpha+1)}\right]$

(3) $h(t) \quad = \quad \dfrac{K\,\beta(\alpha+1)}{(1+\beta t)^{\alpha+2}} \quad = \quad \dfrac{d\,\mu(t)}{d\,t}$

(4) $\rho(t) \quad = \quad \lambda_A + (1 - \mu_d)K\,\beta(\alpha+1) + \dfrac{\mu_d\,K\,\beta(\alpha+1)}{(1+\beta t)^{\alpha+2}}$

(5) $\theta(t) \quad = \quad 1 - (1 + \beta t)^{-(\alpha+2)}$

To show (1), recall $\lambda_{B,K} = E\left(\sum_{i=1}^{K} \Lambda_i\right)$ where $\underline{\Lambda} = (\Lambda_1, \cdots, \Lambda_K)$ is a random sample from Λ. Thus

$$\lambda_{B,K} \quad = \quad K\,E(\Lambda) \quad = \quad K \left.\frac{d\,\Psi(x)}{d\,x}\right|_{x=0}$$

$$= \quad K\,\beta(\alpha+1)$$

To demonstrate (2), note by Annex 1

$$\mu(t) \quad = \quad E[\mu(t;\underline{\Lambda})]$$

$$= \quad E\left[\sum_{i=1}^{K}\left(1 - e^{-\Lambda_i t}\right)\right]$$

$$= \quad K - E\left\{\sum_{i=1}^{K} e^{-\Lambda_i t}\right\}$$

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$$= \quad K - K\,E\left[e^{-\Lambda t}\right]$$

Thus

$$\begin{aligned}
\mu(t) &= K\left\{1 - E\left[e^{-\Lambda t}\right]\right\} \\
&= K\{1 - \Psi(-t)\} \\
&= K\left\{1 - (1 + \beta\, t)^{-(\alpha+1)}\right\}
\end{aligned}$$

To derive (3) we can utilize the expression for $h(t; \underline{\lambda})$ in Annex 1. Doing so we arrive at

$$\begin{aligned}
h(t) &= E\left[h(t; \underline{\Lambda})\right] \\
&= E\left[\sum_{i=1}^{K} \Lambda_i e^{-\Lambda_i t}\right] \\
&= K\,E\left[\Lambda e^{-\Lambda t}\right]
\end{aligned}$$

Note

$$\begin{aligned}
E\left[\Lambda e^{-\Lambda t}\right] &= \left.\frac{d\Psi}{dx}\right|_{x = -t} \\
&= \frac{\beta(\alpha + 1)}{(1 + \beta\, t)^{\alpha+2}}
\end{aligned}$$

This yields

$$h(t) = \frac{K\,\beta(\alpha + 1)}{(1 + \beta\, t)^{\alpha+2}}$$

Note by (2) above,

$$\frac{d\,\mu(t)}{d\,t} = \frac{K\,\beta(\alpha + 1)}{(1 + \beta\, t)^{\alpha+2}}$$

Thus, as expected,

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$$\frac{d\,\mu(t)}{d\,t} \quad = \quad E[h(t;\Lambda)] \quad = \quad h(t)$$

To obtain (4) we recall the expression in (16) of Section 4.4.3 for $\rho(t)$:

$$\rho(t) \quad = \quad \lambda_A + (1-\mu_d)\lambda_{B,K} + \mu_d\, h(t)$$

Thus (4) directly follows from (1) and (3) above.

Finally, recall by (23) of Section 4.4.3 we have

$$\theta(t) \quad = \quad \frac{\lambda_{B,K} - h(t)}{\lambda_{B,K}}$$

By (1) and (3) above we note

$$h(t) \quad = \quad \frac{\lambda_{B,K}}{(1+\beta\, t)^{\alpha+2}}$$

Thus

$$\theta(t) \quad = \quad \frac{\lambda_{B,K} - \dfrac{\lambda_{B,K}}{(1+\beta\, t)^{\alpha+2}}}{\lambda_{B,K}}$$

$$= \quad 1-(1+\beta\, t)^{-(\alpha+2)}$$

Annex 3

Maximum Likelihood Estimates for AMPM

To obtain maximum likelihood estimates (mle's) for our finite K and NHPP variants of the AMPM, assume m distinct B-modes first occur at test times $0 < t_1 \le t_2 \le \cdots \le t_m$ respectively over a test period of length T. Let n_A denote the number of A-mode failures that occur over test period T. We shall denote an estimate of a model parameter by placing the symbol "^" over the parameter. Thus, e.g., $\hat{\lambda}_A = n_A / T$ since λ_A is constant over test period T.

Let $\underset{\sim}{t}$ be the vector of B-mode first occurrence times $(t_1, \cdots, t_m)$. Also, let $\langle K \rangle$ denote the set of positive integers less than or equal to K and let S_m denote the set of all subsets of $\langle K \rangle$ of size m. Then, conditioned on $\underline{\Lambda} = \underline{\lambda}$, the likelihood function for the test data $(m, \underline{t})$ is $L(m, \underline{t}; \underline{\lambda})$ where

$$L(m,\underline{t};\underline{\lambda}) \quad = \quad \sum_{S \in S_m} \left[\prod_{i \in S} \lambda_i \, e^{-\lambda_i t_i} \prod_{i \in \langle K \rangle - S} e^{-\lambda_i T} \right] \tag{1}$$

Thus the unconditional likelihood function for test data $(m, \underline{t})$ is

$$L(m,\underline{t}) \quad = \quad \binom{k}{m} \left[\prod_{i=1}^{m} E\left(\Lambda e^{-\Lambda t_i}\right) \right] \left[E\left(e^{-\Lambda T}\right) \right]^{K-m} \tag{2}$$

where $\Lambda \sim \Gamma(\alpha, \beta)$ and the expectation is with respect to Λ.

By direct calculation of $E\left[\Lambda^p e^{u\Lambda}\right]$, recalling the form of density function f_Λ given in Section 4.4.2, we can show

$$E\left[\Lambda^p e^{u\Lambda}\right] \quad = \quad \frac{(\alpha + p)!\, \beta^p}{\alpha!\,(1 - \beta u)^{\alpha+1+p}} \tag{3}$$

for $u < \beta^{-1}$ and $\beta > -(1+\alpha)$. From (2) and (3) we obtain

$$m!\,L(m,\underline{t}) \quad = \quad K(K-1)\cdots(K-m+1) \left[\frac{\beta^m (\alpha+1)^m}{(1+\beta T)^{(\alpha+1)(K-m)}} \right] \prod_{i=1}^{m} (1 + \beta t_i)^{-(\alpha+2)} \tag{4}$$

Let $Z = \ln\{m!\,L(m,\underline{t})\}$. Then it follows that

$$\frac{\partial Z}{\partial \alpha} \quad = \quad \frac{m}{\alpha + 1} - (K-m)\ln(1+\beta T) - \sum_{i=1}^{m} \ln(1 + \beta t_i) \tag{5}$$

and

$$\frac{\partial Z}{\partial \beta} \quad = \quad \frac{m}{\beta} - \frac{(\alpha+1)(K-m)T}{1+\beta T} - (\alpha+2)\sum_{i=1}^{m} \frac{t_i}{1+\beta t_i} \tag{6}$$

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Treating K as a positive real number we also obtain

$$\frac{\partial Z}{\partial K} = \sum_{i=0}^{m-1} \frac{1}{K-i} - (\alpha+1)\ln(1+\beta T)$$

(7)

In Section 4.4 and this appendix we shall not use (7) since we are only interested in obtaining mle's $\hat{\alpha}$, $\hat{\beta}$ in terms of K and the test data. We shall then hold the test data constant and let $K \to \infty$ to study the limiting behavior of our AMPM estimators. For each K, let $\hat{\alpha}_K$ and $\hat{\beta}_K$ denote the values of α and β respectively that maximize $L(m,\underline{t})$, or equivalently, $Z = \ln(m!L(m,\underline{t}))$. Let $v_K \underline{\Delta} (\alpha, \beta, K)$ and $\hat{v}_K \underline{\Delta} \left(\hat{\alpha}_K, \hat{\beta}_K, K\right)$.

Then by (5) our maximum likelihood equation for α is

$$\left.\frac{\partial Z}{\partial \alpha}\right|_{v_K = \hat{v}_K} = 0 \Leftrightarrow \left(\hat{\alpha}_K + 1\right)^{-1} = m^{-1}\left[K \ln\left(1+\hat{\beta}_K T\right) - \sum_{i=1}^{m} \ln\left(\frac{1+\hat{\beta}_K T}{1+\hat{\beta}_K t_i}\right)\right]$$

(8)

By (6) our maximum likelihood equation for β is

$$\left.\frac{\partial Z}{\partial \beta}\right|_{v_K = \hat{v}_K} = 0 \Leftrightarrow \hat{\alpha}_K + 1 = \frac{\dfrac{m}{\hat{\beta}_K} - \displaystyle\sum_{i=1}^{m} \frac{t_i}{1+\hat{\beta}_K t_i}}{\dfrac{(K-m)T}{1+\hat{\beta}_K T} + \displaystyle\sum_{i=1}^{m} \frac{t_i}{1+\hat{\beta}_K t_i}}$$

(9)

Equating the expressions for $\left(\hat{\alpha}_K + 1\right)^{-1}$ obtained from (8) and (9) we arrive at a linear equation for K. Solving for K we obtain

$$K = \frac{\displaystyle\sum_{i=1}^{m} \ln \frac{1+\hat{\beta}_K T}{1+\hat{\beta}_K t_i} \sum_{i=1}^{m} \frac{1}{1+\hat{\beta}_K t_i} - \frac{m\hat{\beta}_K}{1+\hat{\beta}_K T} \sum_{i=1}^{m} \frac{T-t_i}{1+\hat{\beta}_K t_i}}{\ln\left(1+\hat{\beta}_K T\right) \displaystyle\sum_{i=1}^{m} \frac{1}{1+\hat{\beta}_K t_i} - \frac{m\hat{\beta}_K}{1+\hat{\beta}_K T} T}$$

(10)

For a given K and data set $(m, \underline{t})$ generated over test period T we can solve (10) for $\hat{\beta}_K$. Then we can use either (8) or (9) to obtain $\hat{\alpha}_K$. Using $\left(\hat{\alpha}_K, \hat{\beta}_K, K\right)$ we can estimate all our finite K AMPM projection quantities where $\hat{\lambda}_A = {n_A}/{T}$ and :d is assessed as

$$\pi_d^* = \frac{1}{m} \sum_{i \in obs} d_i^* \qquad (11)$$

In (11), d_i^* will often be based largely on engineering judgement. The value of d_i^* should reflect several considerations: (1) how certain we are that the problem has been correctly identified; (2) the nature of the fix, e.g., its complexity; (3) past FEF experience and (4) any germane testing (including assembly level testing).

In practice, we do not know the value of K. We could try to develop an mle for K based on (7) or by directly maximizing Z. We have found that a solution $\left(\hat{\alpha}, \hat{\beta}, \hat{K}\right)$ to the maximum likelihood equations (5), (6) and (7) can be a saddle point of $L(m, \underline{t})$. This can occur even for a large data set that appears to fit the model well. We present graphs in Section 4.4.6 for such a data set that clearly illustrate the difficulty in obtaining a reasonable estimate for K. Thus we prefer to take the point of view that we should not attempt to assess K. However, by conducting a standard failure modes and effects criticality analysis (FMECA), we can place a lower bound on K, say K_ℓ. Our experience with the AMPM is that if K is substantially higher than m, say, e.g., $K \geq 10m$, then our AMPM projection quantities will be insensitive to the value of K. We believe for a complex system or subsystem it will often be the case that $K_\ell \geq 10m$ or at least the unknown value of K will be $10m$ or higher. The factor of 10 may be larger than necessary. In practice, we suggest exercising the AMPM model with several plausible lower bound values for K and comparing the associated projections with those obtained in the limit as $K \to \infty$. This is illustrated for a data set in Section 4.4.6.

We shall now consider the behavior of our AMPM estimators as $K \to \infty$. To do so, let $\left\langle \hat{\beta}_K \right\rangle_{K>m}$ be a sequence satisfying (10) with limit $\hat{\beta}_\infty \in [0, \infty)$. We shall assume that such a sequence exists for our data set $(m, \underline{t})$ generated over $[0, T]$. Then by (10) we have

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$$\ln\left(1+\hat{\beta}_\infty T\right)\sum_{i=1}^{m}\frac{1}{1+\hat{\beta}_\infty t_i} - \frac{m\hat{\beta}_\infty T}{1+\hat{\beta}_\infty T} \quad = \quad 0$$

(12)

Recall by Annex 2, $\lambda_{B,K} = K\,\beta(\alpha+1)$, where we previously suppressed the subscript K. Thus we shall define $\hat{\lambda}_{B,K}$ by

$$\hat{\lambda}_{B,K} \quad \underline{\Delta} \quad K\hat{\beta}\left(\hat{\alpha}+1\right)$$

(13)

By (8) we obtain

$$\hat{\lambda}_{B,K} \quad = \quad K\hat{\beta}_K\left(\hat{\alpha}_K+1\right) \quad = \quad \frac{K\,m\,\hat{\beta}_K}{K\ln\left(1+\hat{\beta}_K T\right)-\sum_{i=1}^{m}\ln\frac{1+\hat{\beta}_K T}{1+\hat{\beta}_K t_i}}$$

(14)

Taking the limit in (14) as $K\to\infty$ we arrive at

$$\hat{\lambda}_{B,\infty} \quad \underline{\Delta} \quad \lim_{K\to\infty}\hat{\lambda}_{B,K} = \frac{m\hat{\beta}_\infty}{\ln\left(1+\hat{\beta}_\infty T\right)}$$

(15)

provided $\hat{\beta}_\infty > 0$. If $\hat{\beta}_\infty = 0$, then we can show, by applying L'Hospital's rule, that the limit of the right hand side of (10) goes to a finite positive number as $K\to\infty$. This contradiction establishes that $\hat{\beta}_\infty > 0$. Since $K\hat{\beta}_K\left(\hat{\alpha}_K+1\right)\to\hat{\lambda}_{B,\infty}\in(0,1)$ as $K\to\infty$ and $\hat{\beta}_\infty\in(0,\infty)$, we obtain

$$\hat{\alpha}_\infty \quad = \quad \lim_{K\to\infty}\hat{\alpha}_K \quad = \quad -1$$

(16)

We can now obtain our limiting AMPM estimates as $K\to\infty$. We first numerically solve (12) for $\hat{\beta}_\infty$ and then obtain $\hat{\lambda}_{B,\infty}$ from (15). From (16), the value of

$\hat{\alpha}_\infty$ is –1. To go from the finite K AMPM estimate to the associated limiting estimate, we first consider $h_K(t)$ given by (3) in Annex 2, where we have suppressed the subscript K. Motivated by (3), we define

$$\hat{h}_K(t) \;\underline{\Delta}\; \frac{K\hat{\beta}_K\left(\hat{\alpha}_K+1\right)}{\left(1+\hat{\beta}_K t\right)^{\hat{\alpha}_K+2}} \tag{17}$$

Then

$$\hat{h}_\infty(t) \;\underline{\Delta}\; \lim_{K\to\infty}\hat{h}_K(t) \;=\; \frac{\hat{\lambda}_{B,\infty}}{1+\hat{\beta}_\infty t} \tag{18}$$

From (2) in Annex 2, we define

$$\hat{\mu}_K(t) \;\underline{\Delta}\; K\left[1-\left(1+\hat{\beta}_K t\right)^{-\left(\hat{\alpha}_K+1\right)}\right] \tag{19}$$

We can obtain $\hat{\mu}_\infty(t)$ more readily from (18) than from (19).

$$\hat{\mu}_\infty(t) \;\underline{\Delta}\; \lim_{K\to\infty}\hat{\mu}_K(t) \;=\; \lim_{K\to\infty}\int_0^t \hat{h}_K(x)dx \;=\; \int_0^t \lim_{K\to\infty}\hat{h}_K(x)dx$$

$$= \int_0^t \frac{\hat{\lambda}_{B,\infty}\,dx}{\left(1+\hat{\beta}_\infty x\right)} \;=\; \frac{\hat{\lambda}_{B,\infty}}{\hat{\beta}_\infty}\ln\left(1+\hat{\beta}_\infty t\right) \tag{20}$$

From (20), we can see that Equation (15) simply says

$$\hat{\mu}_\infty(T) \;=\; m \tag{21}$$

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In accordance with (5) in Annex 2, we define

$$\hat{\theta}_K(t) \;\underline{\Delta}\; 1-\left(1+\hat{\beta}_K t\right)^{-\left(\hat{\alpha}_K+2\right)}$$

(22)

Then

$$\hat{\theta}_\infty(t) \;\underline{\Delta}\; \lim_{K\to\infty}\hat{\theta}_K = 1-\left(1+\hat{\beta}_\infty t\right)^{-1} = \frac{\hat{\beta}_\infty t}{1+\hat{\beta}_\infty t}$$

(23)

Finally, from (4) in Annex 2, we define

$$\hat{\rho}_K(t) \;\underline{\Delta}\; \hat{\lambda}_A+\left(1-\mu_d^*\right)K\hat{\beta}_K\left(\hat{\alpha}_K+1\right)+\mu_d^*\frac{K\hat{\beta}_K\left(\hat{\alpha}_K+1\right)}{\left(1+\hat{\beta}_K t\right)^{\hat{\alpha}_K+2}}$$

(24)

From (24) we have

$$\hat{\rho}_\infty(t) \;\underline{\Delta}\; \lim_{K\to\infty}\hat{\rho}_K(t) = \hat{\lambda}_A+\left(1-\mu_d^*\right)\hat{\lambda}_{B,\infty}+\mu_d^*\left(\frac{\hat{\lambda}_{B,\infty}}{1+\hat{\beta}_\infty t}\right)$$

(25)

Recall in Section 4.4.4 we showed our finite K AMPM converged to a NHPP in the sense that the process $\{X_K(t), 0\le t<\infty\}$ converged to the NHPP $\{X_\infty(t), 0\le t<\infty\}$ as $K\to\infty$. We also noted that $\{X_\infty(t), 0\le t<\infty\}$ has the mean value function $\mu_\infty(t)$ given in (4.4.4). We could directly derive parameter estimators for this NHPP. By so doing, one can show that these estimators are identical to the limiting AMPM estimators.